# **Digital Image Processing**

### **Image Compression**



Caution: The PDF version of this presentation will appear to have errors due to heavy use of animations

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Material in this presentation is largely based on/derived from presentation(s) and book: The Digital Image by Dr. Donald House at Texas A&M University

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# Lecture Objectives

- Previously
  - Filtering
  - Interpolation
  - Warping
  - Morphing

 Image Manipulation and Enhancement

- Today
  - Image Compression

# **Definition: File Compression**

- Compression: the process of encoding information in fewer bits
  - Wasting space is bad, so compression is good
  - Image Compression
    - Redundant information in images
      - Identical colors
      - Smooth variation in light intensity
      - Repeating texture

## **Identical Colors**



Mondrian's Composition 1930

## Smooth Variation in Light Intensity



Digital rendering using Autodesk VIZ. (Image Credit: Alejandro Vazquez.)

# **Repeating Texture**





Alvar Aalto Summer House 1953

# What is Compression Really?

- Works because of data redundancy
  - Temporal
    - In 1D data, 1D signals, Audio...
  - Spatial
    - correlation between neighboring pixels or data items
  - Spectral
    - correlation between color or luminescence components
    - uses the frequency domain to exploit relationships between frequency of change in data
  - Psycho-visual
    - exploits perceptual properties of the human (visual) system

# Two General Types

- Lossless Compression
  - data is compressed and can be uncompressed without loss of detail or information
    - bit-preserving
    - reversible
- Lossy Compression
  - purpose is to obtain the best possible fidelity for a given bit-rate
    - or minimizing the bit-rate to achieve a given fidelity measure
  - Video and audio commonly use lossy compression
    - because humans have limited perception of finer details

# Two Types

- Lossless compression often involves some form of entropy encoding
  - based in information theoretic techniques
    - see next slide for visual
- Lossy compression uses source encoding techniques that may involve transform encoding, differential encoding or vector quanatization
  - see next slide for visual

### **Compression Methods**



### **Compression Methods**



# Simple Lossless Compression

- Simple Repetition Suppression
  - If a sequence contains a series of N successive tokens
  - Then they can be replaced with a single token and a count of the number of times the token repeats
    - This does require a flag to denote when the repeated token appears
  - Example
    - 12344444444
    - can be denoted
    - 123f9
    - where f is the flag for four

# Run-length Encoding (RLE)

- RLE is often applied to images
  It is a small component used in JPEG compression
- Conceptually
  - sequences of image elements X<sub>1</sub>, X<sub>2</sub>,...,X<sub>n</sub> are mapped to pairs (c<sub>1</sub>, L<sub>1</sub>), (c<sub>2</sub>, L<sub>2</sub>),...,(c<sub>n</sub>, L<sub>n</sub>)
    - where c<sub>i</sub> represent the image intensity or color
    - and  $L_i$  the length of the i<sup>th</sup> run of pixels

# Run-Length Encoding (RLE): lossless

• Scanline: 2 2 2 2 2 2 2 3 4 1 1 1 12 values

Run-length encoding
 (7 2) (1 3) (1 4) (3 1)

8 values



25% reduction of memory use

### RLE: worst case

• Scanline: 1 2 3 4 5 6 7 8

8 values

### Run-length encoding: (11)(12)(13)(14)(15)(16)(17)

doubles space

# **RLE: Improving**

• Scanline: 5 5 5 5 5 5 5 3 4 1 1 1



# How to flag the repeat/no repeat?

• SGI Iris RGB Run-length Encoding Scheme

	n		valu	le
7		0	7	0

 $1 \le n \le 127$ , Repeat count for run of length = n



 $128 \le n \le 255, n-128$  gives number of nonrepeating values that follow



### **Compression Methods**



# **Compression: Pattern Substitution**

• Pattern Substitution, lossless

• Simple form of statistical encoding

- Concept
  - Substitute a frequently repeating pattern with a shorter code
    - the shorter code(s) may be predefined by the algorithm being used or dynamically created

# Table Lookup

 Table Lookup can be viewed as a Pattern Substitution Method

- Example
  - Allow full range of colors (24 bit, RGB)
  - Image only uses 256 (or less) unique colors (8 bits)
  - Create a table of which colors are used
    - Use 8 bits for each color instead of 24 bits
      - Conceptually how older BMPs worked
        - » color depth <= 8 bits</p>

# Table Lookup: GIF

- Graphics Interchange File Format (GIF)
  - − uses table lookups  $\rightarrow$  Color LookUp Table (CLUT)



#### Figure 3.8: 8-Bit Color Framebuffer with 3 Lookup Tables

### GIF Compression with Color LookUp Table (CLUT)



#### Example

Image Size = 1000x1000 256 colors Each color 24 bit (RGB)

without CLUT 1000\*1000\*24 bits

with CLUT 1000\*1000\*8 bit (index data) + 3\*256\*8bit (table data)

Use about 2/3 the space (when image size is "big")

#### Figure 3.8: 8-Bit Color Framebuffer with 3 Lookup Tables

# **Compression: Pattern Substitution**

• Table lookups work

- But Pattern Substitution typically is more dynamic
  - Counts occurrence of tokens
  - Sorts (say descending order)
  - Assign highest counts shortest codes

### **Compression Methods**



### Lossless Compression: Entropy Encoding

 Lossless compression often involves some form of entropy encoding and are based in information theoretic techniques

- Aside:
  - Claude Shannon is considered the father of information theory

# Shannon-Fano Algorithm

- Technique proposed in Shannon's 1948 article. introducing the field of Information Theory:
  - A Mathematical Theory of Communication
    - Shannon, C.E. (July 1948). "A Mathematical Theory of Communication". Bell System Technical Journal 27: 379–423.
- Method Attributed to Robert Fano, as published in a technical report
  - The transmission of information
    - Fano, R.M. (1949). "The transmission of information". Technical Report No. 65 (Cambridge (Mass.), USA: Research Laboratory of Electronics at MIT).

Symbol	А	В	С	D	E
Count	15	7	6	6	5

Symbol	E	В	А	D	С
Count	15	7	6	6	5

Step 1: Sort the symbols by frequency/probability As shown: (



Symbol	E	В	А	D	С
Count	15	7	6	6	5

Step 1: Sort the symbols by frequency/probability

Step 2: Recursively divide into 2 parts Each with about same number of counts

> Dividing between B and A results in 22 on the left and 17 on the right -- minimizing difference totals between groups

This division means E and B codes start with 0 and A D and C codes start with 1



Symbol	E	В	A	D	С
Count	15	7	6	6	5

Step 1: Sort the symbols by frequency/probability

Step 2: Recursively divide into 2 parts Each with about same number of counts

> Dividing between B and A results in 22 on the left and 17 on the right -- minimizing difference totals between groups

This division means E and B codes start with 0 and A D and C codes start with 1

E and B are then divided (15:7) A is divided from D and C (6:11)

> So E is leaf with code 00, B is a leaf with code 01 A is a leaf with code 10 D and C need divided again

As shown: E B A D C E B A D C E B A D C E B A D CE B A D C

Symbol	E	В	А	D	С
Count	15	7	6	6	5

Step 1: Sort the symbols by frequency/probability

Step 2: Recursively divide into 2 parts Each with about same number of counts

> So E is leaf with code 00, B is a leaf with code 01 A is a leaf with code 10 D and C need divided again

Divide D and C (6:5)

D becomes a leaf with code 110 C becomes a leaf with code 111



Symbol	E	В	A	D	С
Count	15	7	6	6	5

Step 1: Sort the symbols by frequency/probability

Step 2: Recursively divide into 2 parts Each with about same number of counts

#### **Final Encoding:**

Symbol	E	В	А	D	С
Count	00	01	10	110	111



### **Compression Methods**



# Quick Summary: Huffman Algorithm

### • Encoding Summary

Step 1: Initialization

Put all nodes in an OPEN list (keep it sorted at all times)

Step 2: While OPEN list has more than 1 node

- Step 2a: From OPEN pick 2 nodes having the lowest frequency/probability Create a parent node for them
- Step 2b: Assign the sum of the frequencies of the selected node to their newly created parent
- Step 2c: Assign code 0 to the left branch Assign code 1 to the right branch Remove the selected children from OPEN (note the newly created parent node remains in OPEN)

## Observation

- Some characters in the English alphabet occur more frequently than others
  - The table below is based on Robert Lewand's Cryptological Mathematics

Letter 🔺	Relative	frequency in the English language $\Rightarrow$
а	8.167%	
b	1.492%	
c	2.782%	
d	4.253%	
e	12.702%	
f	2.228%	
g	2.015%	
h	6.094%	
i	6.966%	
j	0.153%	
k	0.772%	
I	4.025%	
m	2.406%	
f g h i j k l m	2.228% 2.015% 6.094% 6.966% 0.153% 0.772% 4.025% 2.406%	

Letter 🔺	Relative	frequency in the English language 💠
n	6.749%	
0	7.507%	
р	1.929%	
q	0.095%	
r	5.987%	
S	6.327%	
t	9.056%	
u	2.758%	
v	0.978%	
w	2.360%	
x	0.150%	
у	1.974%	
z	0.074%	
# Huffman Encoding (English Letters)

- **Huffman encoding**: Uses variable lengths for different characters to take advantage of their relative frequencies
  - Some characters occur more often than others
    - If those characters use < 8 bits each, the file will be smaller
  - Other characters may need > 8 bits
    - but that's ok  $\rightarrow$  they don't show up often

Char	ASCII value	ASCII (binary)	Hypothetical Huffman
1 1	32	00100000	10
'a'	97	01100001	0001
'b'	98	01100010	01110100
'c'	99	01100011	001100
'e'	101	01100101	1100
' <sub>Z</sub> '	122	01111010	00100011110

# Huffman's Algorithm

- The idea: Create a "Huffman Tree" that will tell us a good binary representation for each character
  - Left means 0
  - Right means 1
    - Example 'b' is 10
- More frequent characters will be higher in the tree (have a shorter binary value).
- To build this tree, we must do a few steps first
  - Count occurrences of each unique character in the file to compress
  - Use a priority queue to order them from least to most frequent
  - Make a tree and use it



# Huffman Compression – Overview

• Step 1

- Count characters (frequency of characters in the message)

- Step 2
  - Create a Priority Queue
- Step 3
  - Build a Huffman Tree
- Step 4
  - Traverse the Tree to find the Character to Binary Mapping
- Step 5
  - Use the mapping to encode the message

#### Step 1: Count Characters

 Example message (input file) contents: *file ends with an invisible EOF character*

- counts: { ' ' = 2, 'b'=3, 'a' =3, 'c' =1, EOF=1 }

byte	1	2	3	4	5	б	7	8	9	10
char	'a'	'b'	1 1	'a'	'b'	1 1	'C'	'a'	'b'	EOF
ASCII	97	98	32	97	98	32	99	97	98	256
binary	01100001	01100010	00100000	01100001	01100010	00100000	01100011	01100001	01100010	N/A

– File size currently = 10 bytes = 80 bits

#### Step 2: Create a Priority Queue

- Each node of the PQ is a tree
  - The root of the tree is the 'key'
  - The other internal nodes hold 'subkeys'
  - The leaves hold the character values
- Insert each into the PQ using the PQ's function
  - insertItem(count, character)
- The PQ should organize them into ascending order
  - So the smallest value is highest priority
    - We will use an example with the PQ implemented as an ordered list
      - But the PQ could be implemented in whatever way works best
        - » could be a minheap, unordered list, or 'other'

#### Step 2: PQ Creation, An Illustration

- From step 1 we have

   counts: { ' ' = 2, 'b'=3, 'a'=3, 'c'=1, EOF=1 }
- Make these into trees
- Add the trees to a Priority Queue

– Assume PQ is implemented as a sorted list

#### Step 2: PQ Creation, An Illustration

- From ste 2 3 3 1 1 - counts: 1 = 2 0 - 3, a - 3, c - 1, cOF=1 }
- Make the '' b a c EOF
- Add the trees to a Priority Queue

- Assume PQ is implemented as a sorted list

#### Step 2: PQ Creation, An Illustration

- From step 1 we have

   counts: { ' ' = 2, 'b'=3, 'a'=3, 'c'=1, EOF=1 }
- Make these into trees
- Add the trees to a Priority Queue
  - Assume PQ is implemented as a sorted list

Recall: Each Insert is an O(n) operation This is done *n* times. So this is  $O(n^2)$ 



### Step 3: Build the Huffman Tree

- Aside: All nodes should be in the PQ
- While PQ.size() > 1
  - Remove the two highest priority (rarest) nodes
    - Removal done using PQ's removeMin() function
  - Combine the two nodes into a single node
    - So the new node is a tree with
      - root has key value = sum of keys of nodes being combined

Example next slide

- left subtree is the first removed node
- right subtree is the second removed node
- Insert the combined node back into the PQ
- end While
- Remove the one node from the PQ
   This is the Huffman Tree

# Step 3a: Building Huffman Tree, Illus.

• Remove the two highest priority (rarest) nodes



# Step 3b: Building Huffman Tree, Illus.

• Combine the two nodes into a single node





# Step 3c: Building Huffman Tree, Illus.

• Insert the combined node back into the PQ

EOF

1

С



# Step 3d: Building Huffman Tree, Illus.

• PQ has 4 nodes still, so repeat



#### Step 3a: Building Huffman Tree, Illus.

• Remove the two highest priority (rarest) nodes



# Step 3b: Building Huffman Tree, Illus.

• Combine the two nodes into a single node





# Step 3c: Building Huffman Tree, Illus.

• Insert the combined node back into the PQ





# Step 3d: Building Huffman Tree, Illus.

• 3 nodes remain in PQ, repeat again



# Step 3a: Building Huffman Tree, Illus.

• Remove the two highest priority (rarest) nodes



# Step 3b: Building Huffman Tree, Illus.

• Combine the two nodes into a single node





# Step 3c: Building Huffman Tree, Illus.

• Insert the combined node back into the PQ





# Step 3d: Building Huffman Tree, Illus.

• 2 nodes still in PQ, repeat one more time



### Step 3a: Building Huffman Tree, Illus.

• Remove the two highest priority (rarest) nodes



# Step 3b: Building Huffman Tree, Illus.

• Combine the two nodes into a single node



### Step 3c: Building Huffman Tree, Illus.

Insert the combined node back into the PQ



# Step 3d: Building Huffman Tree, Illus.

• Only 1 node remains in the PQ, so while loop ends



# Step 3: Building Huffman Tree, Illus.

• Huffman tree is complete















#### Step 5: Encode the Message

file ends with an invisible EOF character



• 'c' = 010

=

1 1

00

- EOF = 011
- 'b' = 10
- 'a' = 11

#### Challenge: Encode the Message

- '' = 00
- 'c' = 010
- EOF = 011
- 'b' = 10
- 'a' = 11

Example message (input file) contents:
 ab ab cab

file ends with an invisible EOF character




















### Step 5: Encode the Message

- '' = 00
- 'c' = 010
- EOF = 011
- 'b' = 10
- 'a' = 11

• Example message (input file) contents: ab ab cab

file ends with an invisible EOF character

#### • 11<u>10</u>00<u>11</u>10<u>00</u>010<u>11</u>10<u>011</u>

- Count the bits used = 22 bits
- versus the 80
  - previously needed
- File is almost <sup>1</sup>/<sub>4</sub> the size
  - lots of savings

#### Decompression

• From the previous tree shown we now have the message characters encoded as:

char	'a'	'b'	1 1	'a'	'b'	1 1	'C'	'a'	'b'	EOF
binary	11	10	00	11	10	00	010	11	10	011

• Which compresses to bytes 3 like so:

byte	1				2		3		
char	a	b	a	b	С	a	b	EOF	
binary	<u>11</u>	<u>10 (</u>	<u>)0 11</u>	<u>10 (</u>	<u>)0 01</u>	<u>) 1</u>	110	011	

- How to decompress?
  - Hint: Lookup table is not the best answer, what is the first symbol?... 1=? or is it 11? or 111? or 1110? or...

#### Decompression via Tree

- The tree is known to the recipient of the message
  - So use it
- To identify symbols we will Apply the Prefix Property
  - No encoding *A* is the prefix of another encoding *B*
  - Never will have  $x \rightarrow 011$  and  $y \rightarrow 011100110$

#### Decompression via Tree

- Apply the Prefix Property
  - No encoding *A* is the prefix of another encoding *B*
  - Never will have  $x \rightarrow 011$  and  $y \rightarrow 011100110$
- the Algorithm
  - Read each bit one at a time from the input
  - If the bit is 0 go left in the tree
  - Else if the bit is 1 go right
  - If you reach a leaf node
    - output the character at that leaf
    - and go back to the tree root

- Say the encrypted message was:
- 1011010001101011011
  - note: this is NOT the same message as the encryption just done (but the tree the is same)

• Read each bit one at a time

- If it is 0 go left
- If it is 1 go right
- If you reach a leaf, output the character there and go back to the tree root



#### Class Activity: Decompressing Example

- Say the encrypted message was:
- 1011010001101011011
  - note: this is NOT the same message as the encryption just done (but the tree the is same)

• Read each bit one at a time

- If it is 0 go left
- If it is 1 go right
- If you reach a leaf, output the character there and go back to the tree root
  - Pause for students to complete



- Say the encrypted message was:
- 1011010001101011011

• Read each bit one at a time

- If it is 0 go left
- If it is 1 go right
- If you reach a leaf, output the character there and go back to the tree root



- Say the encrypted message was:
  1011010001101011011
  b
  - Read each bit one at a time
  - If it is 0 go left
  - If it is 1 go right
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- Say the encrypted message was:
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## b a c

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- If it is 0 go left
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- Say the encrypted message was:
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# b a c a

- Read each bit one at a time
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- 10<u>11</u>010<u>00</u>1101011011



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- Say the encrypted message was:
- 10<u>11</u>010<u>00</u>11<u>010</u>11011



- Read each bit one at a time
- If it is 0 go left
- If it is 1 go right
- If you reach a leaf, output the character there and go back to the tree root



- Say the encrypted message was:
- 10<u>110100011010</u>11011



- Read each bit one at a time
- If it is 0 go left
- If it is 1 go right
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- Say the encrypted message was:
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Say the encrypted message was:

- Read each bit one at a time
- If it is 0 go left
- If it is 1 go right
- If you reach a leaf, output the character there and go back to the tree root









#### Lempel-Ziv-Welch (LZW) Compression

- Lossless
- Has a table
- Does not store the table
### LZW Compression

- Discovers and remembers **patterns** of colors
- Stores the patterns in a table
  - BUT only table indices are stored in the file

- LZW table entries can grow arbitrarily long,
  - So one table index can stand for a long string of data in the file
  - BUT again the table itself never needs to be stored in the file

## LZW Encoder: Pseudocode

```
initialize TABLE[0 to 255] = code for individual bytes
STRING = get input symbol
while there are still input symbols:
    SYMBOL = get input symbol
    if STRING + SYMBOL is in TABLE:
        STRING = STRING + SYMBOL
    else:
        output the code for STRING
        add STRING + SYMBOL to TABLE
        STRING = SYMBOL
    output the code for STRING
```

### LZW Decoder: Pseudocode

```
initialize TABLE[0 to 255] = code for individual bytes RGB = 3 bytes
CODE = read next code from encoder but idea stays same
STRING = TABLE[CODE]
output STRING
while there are still codes to receive:
   CODE = read next code from encoder
   if TABLE[CODE] is not defined: // needed because sometimes the
      ENTRY = STRING + STRING[0] // decoder may not yet have entry
   else:
      ENTRY = TABLE[CODE]
   output ENTRY
   add STRING+ENTRY[0] to TABLE
   STRING = ENTRY
```

### **Questions?**

- Beyond D2L
  - Examples and information can be found online at:
    - http://docdingle.com/teaching/cs.html

• Continue to more stuff as needed

#### **Extra Reference Stuff Follows**



# Credits

- Much of the content derived/based on slides for use with the book:
  - Digital Image Processing, Gonzalez and Woods
- Some layout and presentation style derived/based on presentations by
  - Donald House, Texas A&M University, 1999
  - Bernd Girod, Stanford University, 2007
  - Shreekanth Mandayam, Rowan University, 2009
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