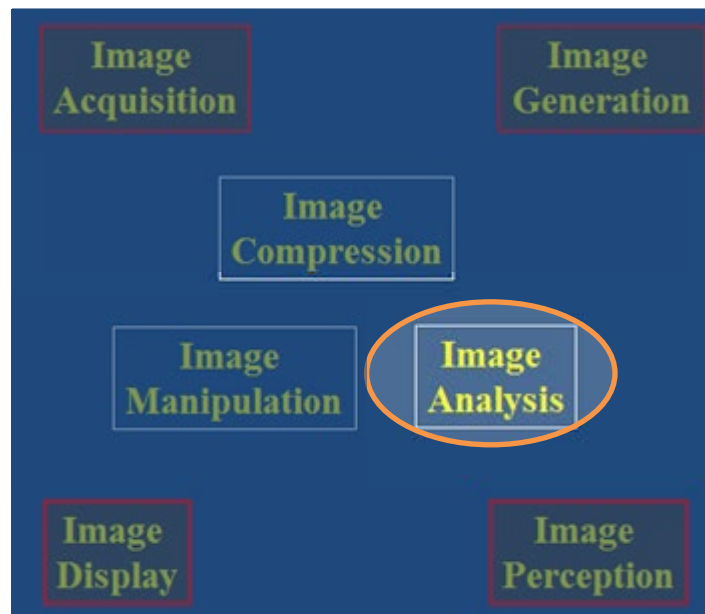


# Digital Image Processing

## Stereo Images



**DRAFT**



# Lecture Objectives

- Previously
  - Image Manipulation and Enhancement
    - Filtering
    - Interpolation
    - Warping
    - Morphing
  - Image Compression
  - Image Analysis
    - Edge Detection
    - Smart Scissors
- Today
  - Stereo Images

# Stereo Images, Relation

- Stereo Image Processing
  - Builds from previous topics
  - Requires methods derived from image manipulation and enhancement
  - Relates to feature detection and extraction
  - Relates to human stereoscopic vision
    - Useful in navigating 3D environments
      - i.e. computer vision for robots



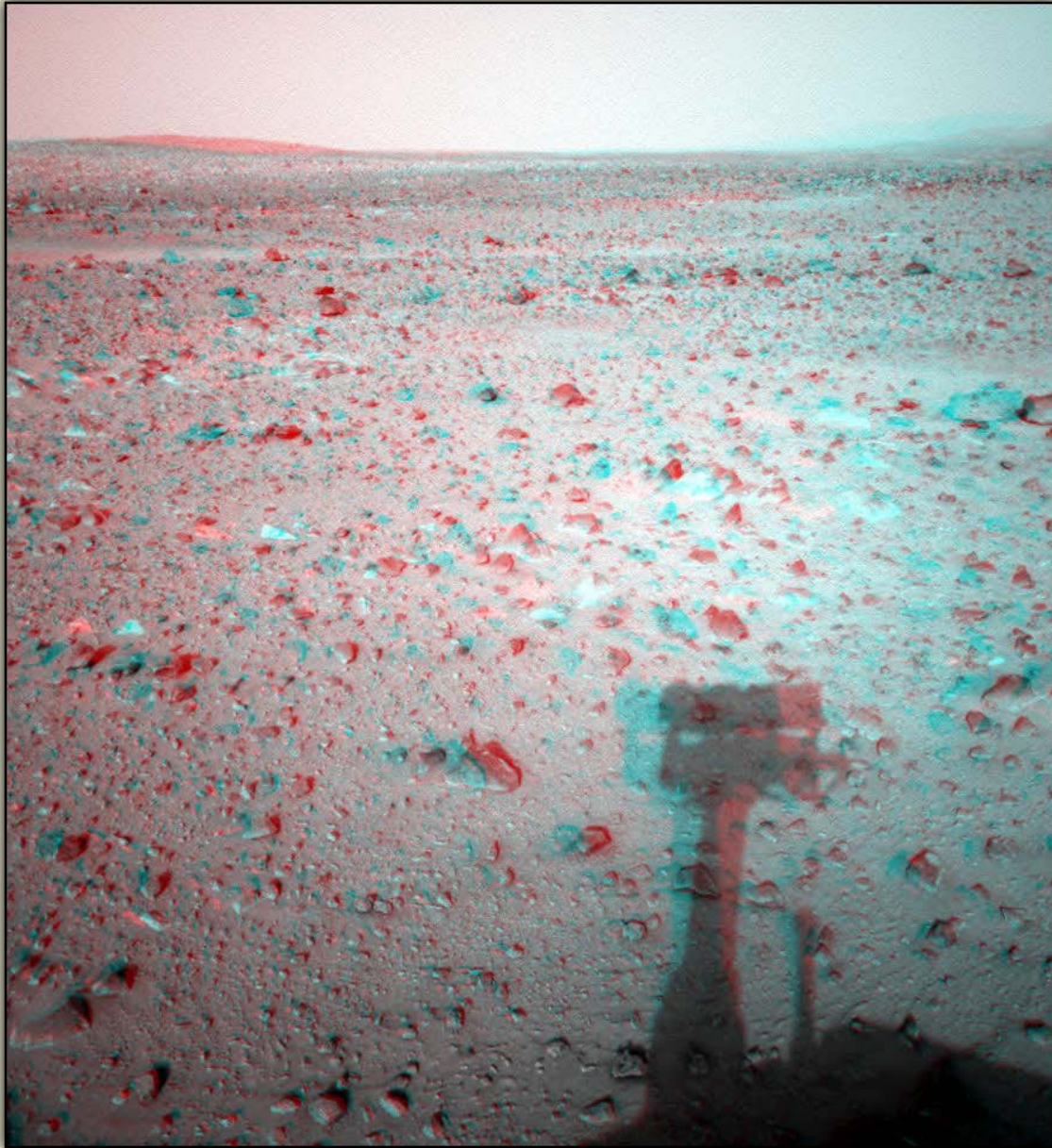
Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923



Mark Twain at Pool Table", no date, UCR Museum of Photography

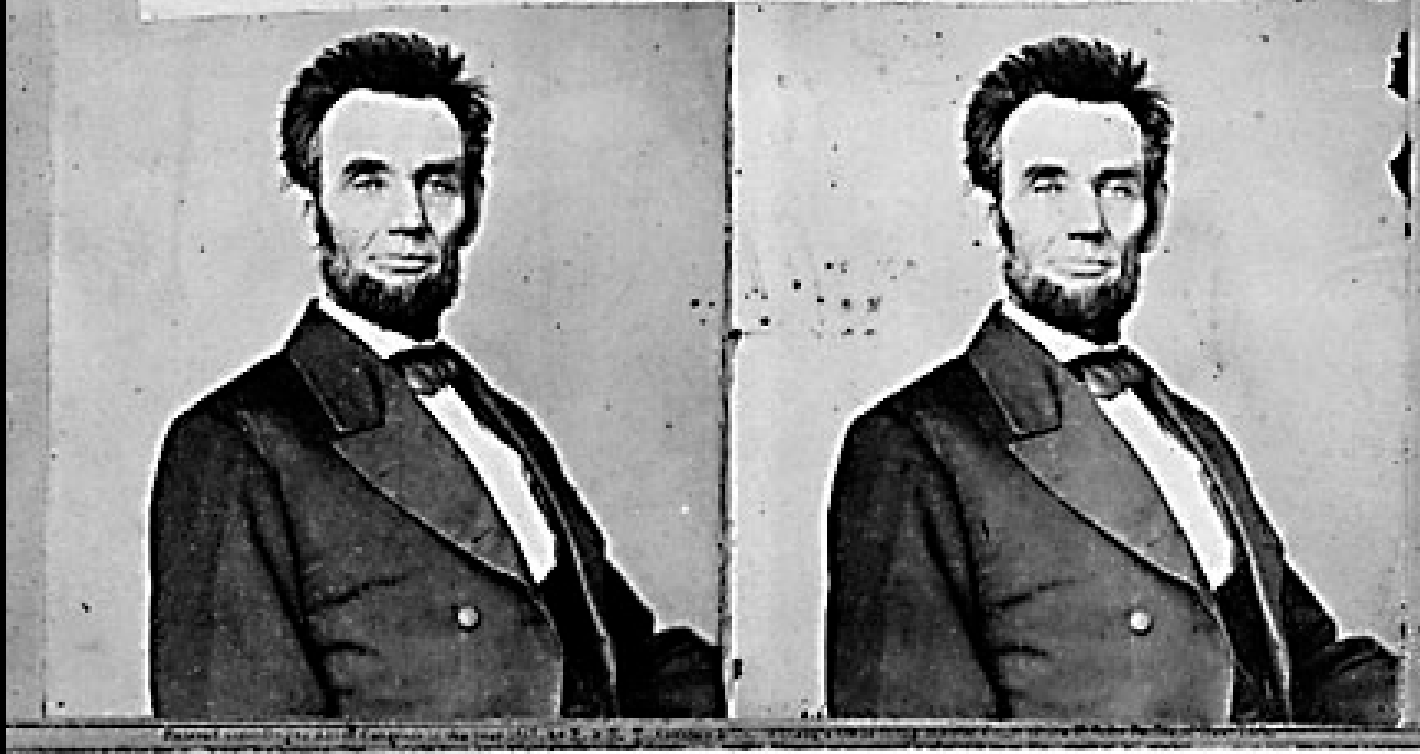
*slide by: Noah Snavely*





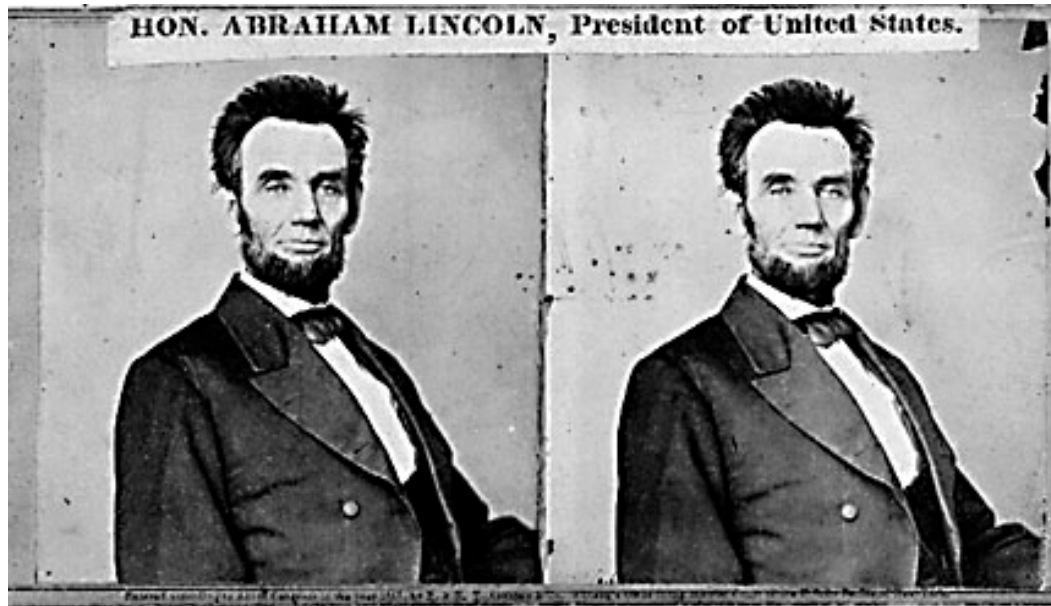
*slide by: Noah Snavely*

HON. ABRAHAM LINCOLN, President of United States.



# Stereo Image Goal

- Given 2 images from different viewpoints
  - Compute the depth of each point in the image
  - Suggested Method
    - Perform the computation based on how much each pixel moves between the two images



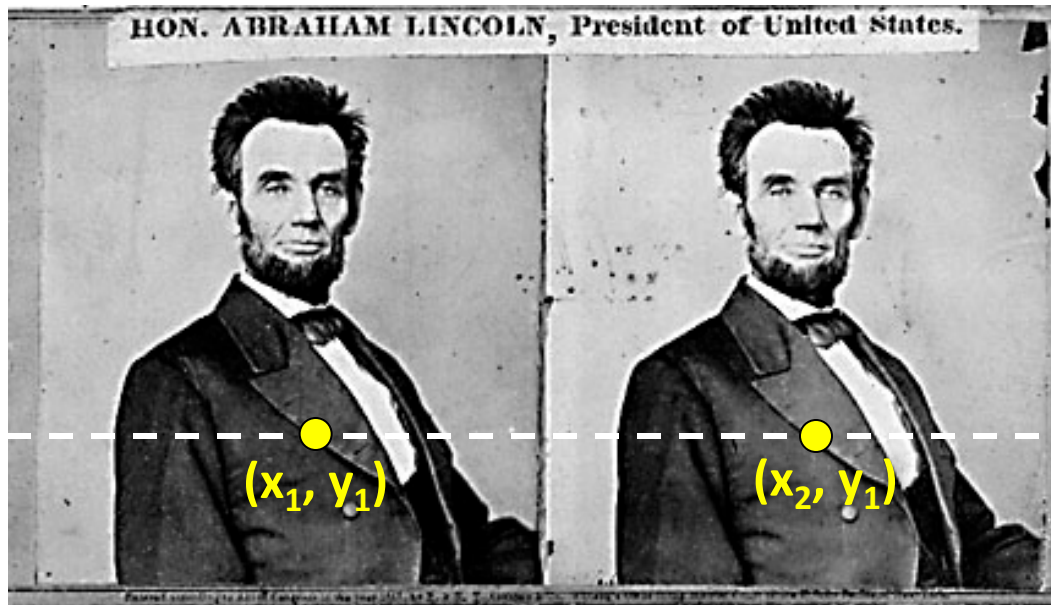


# Epipolar Geometry

- $x_2 - x_1 =$  the disparity of pixel  $(x_1, y_1)$

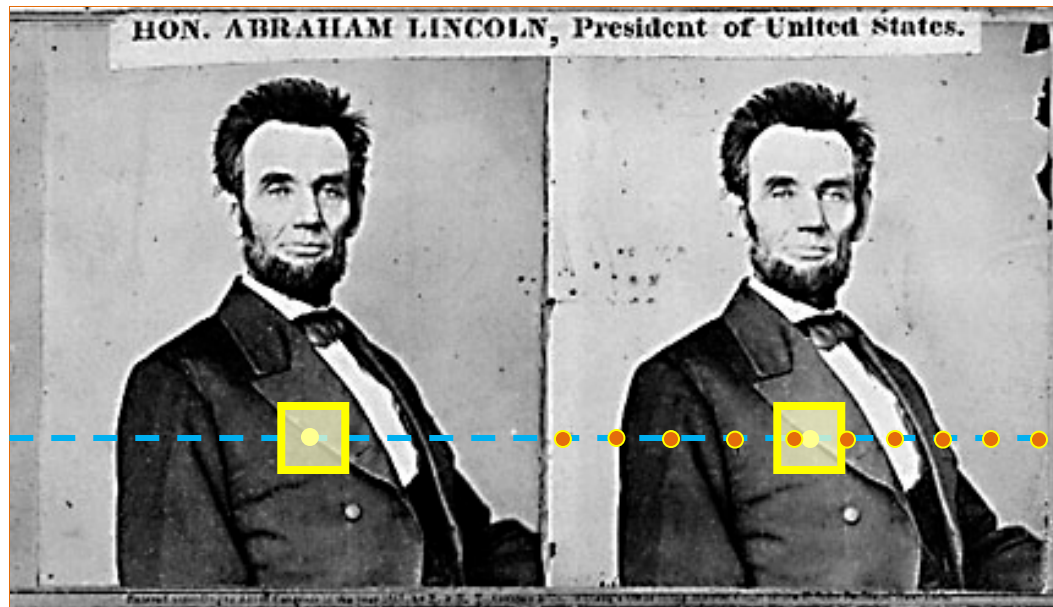
Two images captured by a purely horizontal translating camera  
(*rectified* stereo pair)

epipolar  
lines



# Simple Stereo Algorithm

- For each epipolar line
  - For each pixel in the 'left' image
    - Compare it with every pixel in the same epipolar line of the 'right' image
    - Select the pixel with minimum cost match

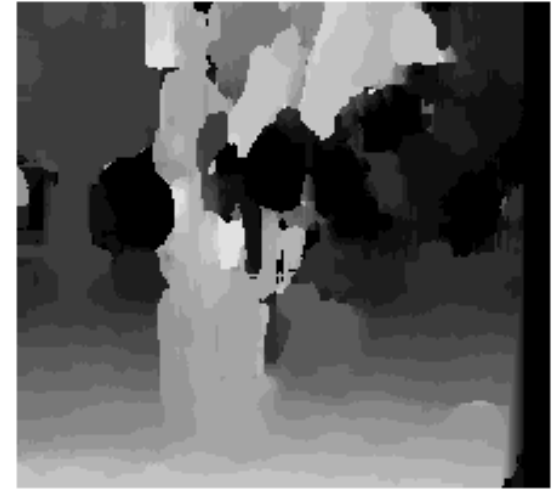


*Possible improvement: use matching windows instead of pixels*

# Window size



$W = 3$



$W = 20$

## Effect of window size

- Smaller window
  - more detail
  - more noise
- Larger window
  - less noise
  - more detail

## Better results with *adaptive window*

- T. Kanade and M. Okutomi, [A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment](#), Proc. International Conference on Robotics and Automation, 1991.
- D. Scharstein and R. Szeliski. [Stereo matching with nonlinear diffusion](#). International Journal of Computer Vision, 28(2):155-174, July 1998

# Stereo results

- Data from University of Tsukuba
- Similar results on other images without ground truth

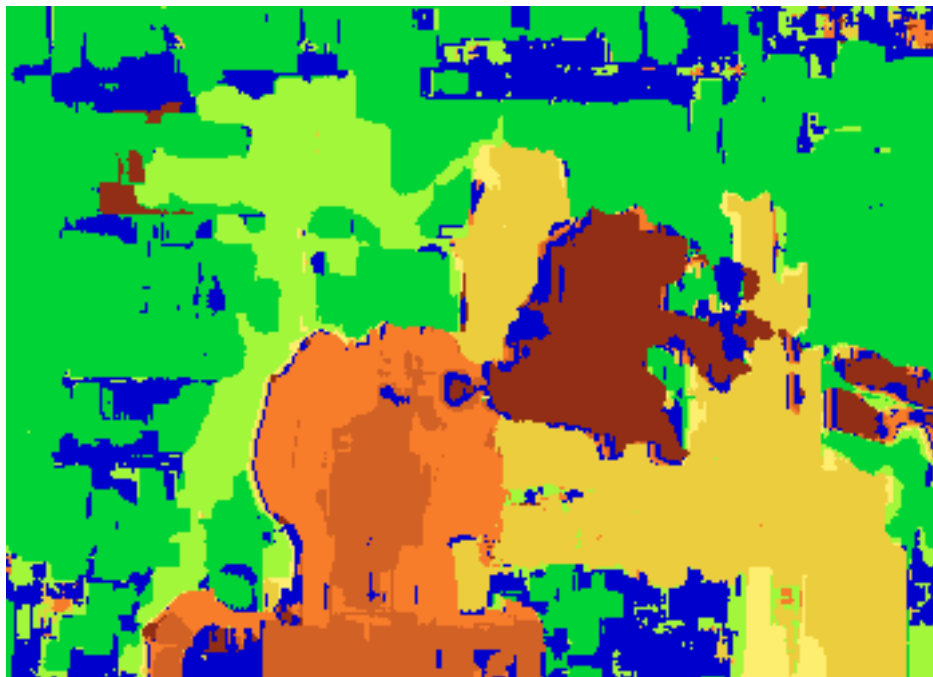


Scene



Ground truth

# Results with window search



Window-based matching  
(best window size)



Ground truth

# Better methods exist...



State of the art method

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](#),  
International Conference on Computer Vision, September 1999.

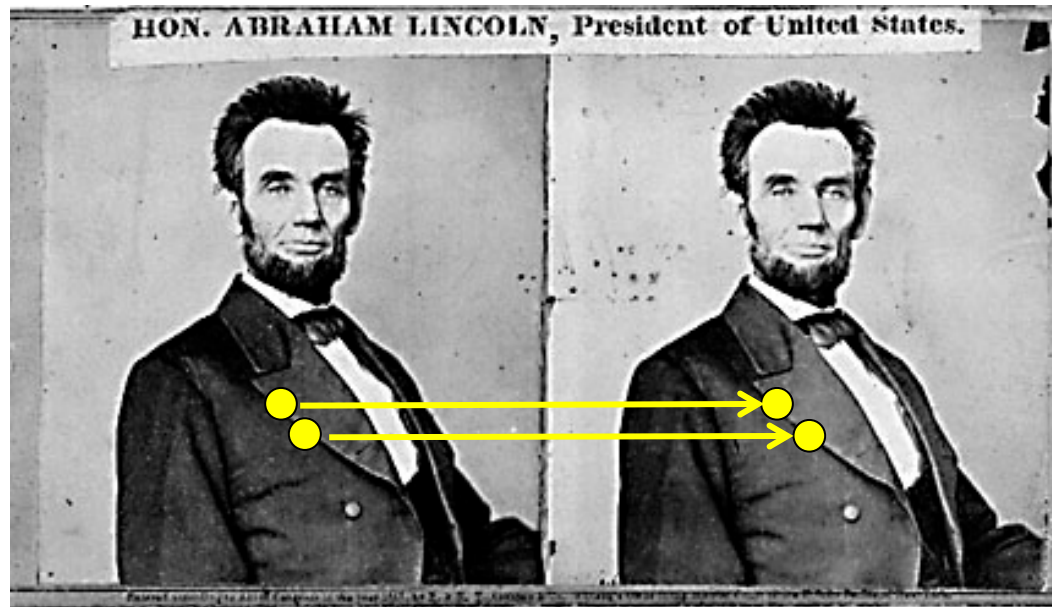


Ground truth



# Stereo as Energy Minimization

- What defines a good stereo correspondence?
  - Match quality
    - Each pixel needs a good match in the other image
  - Smoothness
    - Two adjacent pixels in one image should move about the same amount



# Stereo as Energy Minimization

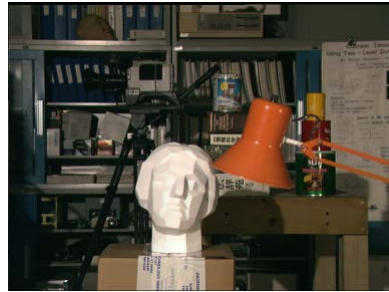
- Find a disparity map  $d$  that minimizes an energy function:  $E(d)$

- Simple pixel / window matching

$$E(d) = \sum_{(x,y) \in I} C(x, y, d(x, y))$$

$$C(x, y, d(x, y)) = \text{SSD distance between windows } I(x, y) \text{ and } J(x + d(x, y), y)$$

# Stereo as energy minimization



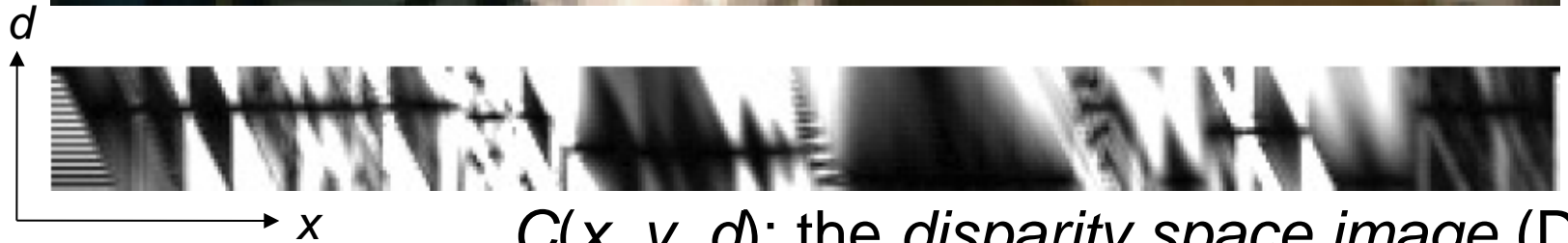
$I(x, y)$



$J(x, y)$



$y = 141$



$C(x, y, d)$ ; the *disparity space image* (DSI)

# Stereo as energy minimization



Simple pixel / window matching: choose the minimum of each column in the DSI independently:

$$d(x, y) = \arg \min_{d'} C(x, y, d')$$

# Stereo as energy minimization

- Better objective function

$$E(d) = \underbrace{E_d(d)}_{\text{match cost}} + \lambda \underbrace{E_s(d)}_{\text{smoothness cost}}$$

Want each pixel to find a good match in the other image

Adjacent pixels should (usually) move about the same amount

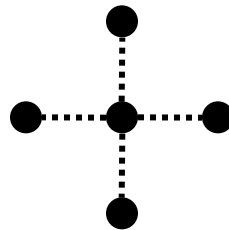
# Stereo as energy minimization

$$E(d) = E_d(d) + \lambda E_s(d)$$

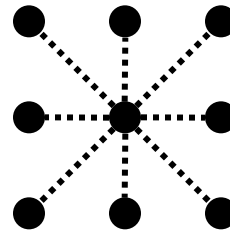
match cost:  $E_d(d) = \sum_{(x,y) \in I} C(x, y, d(x, y))$

smoothness cost:  $E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$

$\mathcal{E}$  : set of neighboring pixels



4-connected neighborhood



8-connected neighborhood



# Smoothness cost

$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$$

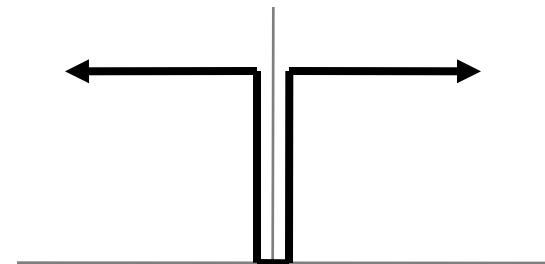
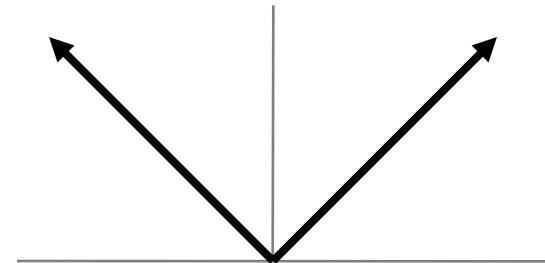
How do we choose  $V$ ?

$$V(d_p, d_q) = |d_p - d_q|$$

$L_1$  distance

$$V(d_p, d_q) = \begin{cases} 0 & \text{if } d_p = d_q \\ 1 & \text{if } d_p \neq d_q \end{cases}$$

“Potts model”



# Dynamic programming

$$E(d) = E_d(d) + \lambda E_s(d)$$

- Can minimize this independently per scanline using dynamic programming (DP) ●.....●.....●

$D(x, y, d)$  : minimum cost of solution such that  $d(x,y) = d$

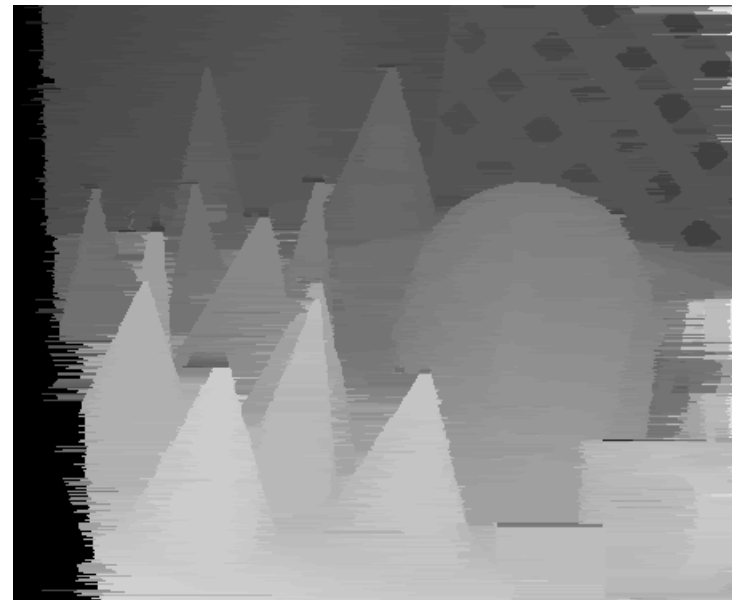
$$D(x, y, d) = C(x, y, d) + \min_{d'} \{D(x - 1, y, d') + \lambda |d - d'|\}$$

# Dynamic programming



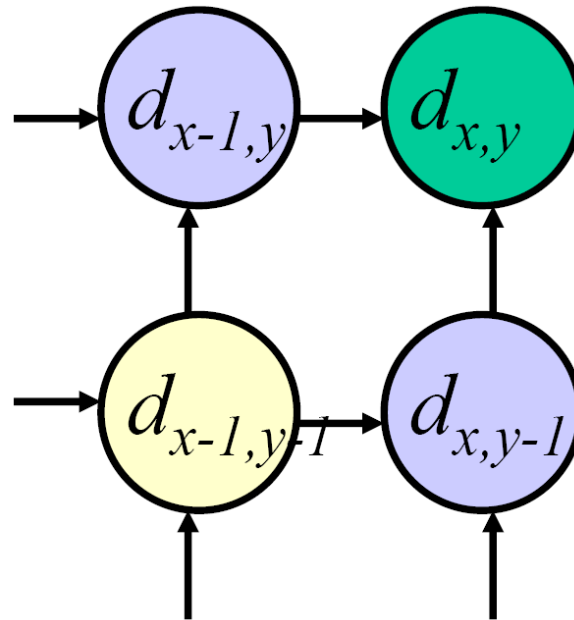
- Finds “smooth” path through DPI from left to right

# Dynamic Programming



# Dynamic programming

- Can we apply this trick in 2D as well?



- No:  $d_{x,y-1}$  and  $d_{x-1,y}$  may depend on different values of  $d_{x-1,y-1}$

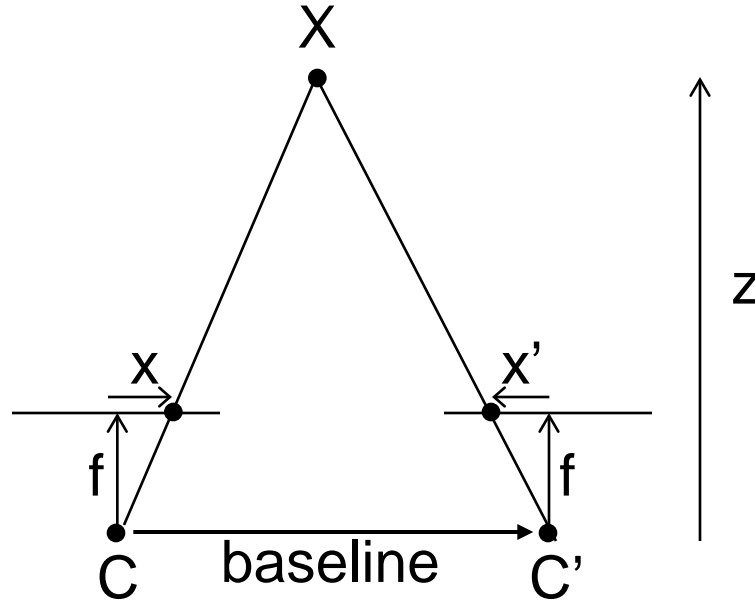
# Stereo as a minimization problem

$$E(d) = E_d(d) + \lambda E_s(d)$$

- The 2D problem has many local minima
  - Gradient descent doesn't work well
- And a large search space
  - $n \times m$  image w/  $k$  disparities has  $k^{nm}$  possible solutions
  - Finding the global minimum is NP-hard in general
- Good approximations exist...



# Depth From Disparity



$$\text{disparity} = x - x' = \frac{\text{baseline} * f}{z}$$

# Stereo reconstruction pipeline

- Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

What will cause errors?

- Camera calibration errors
- Poor image resolution
- Occlusions
- Violations of brightness constancy (specular reflections)
- Large motions
- Low-contrast image regions

# Real-time stereo



[Nomad robot](http://www.frc.ri.cmu.edu/projects/meteorobot/index.html) searches for meteorites in Antarctica  
<http://www.frc.ri.cmu.edu/projects/meteorobot/index.html>

- Used for robot navigation (and other tasks)
  - Several software-based real-time stereo techniques have been developed (most based on simple discrete search)

# Questions?

- Beyond D2L
  - Examples and information can be found online at:
    - *<http://docdingle.com/teaching/cs.html>*
  
- *Continue to more stuff as needed*

Extra Reference Stuff Follows

# Credits

- Much of the content derived/based on slides for use with the book:
  - *Digital Image Processing*, Gonzalez and Woods
- Some layout and presentation style derived/based on presentations by
  - Donald House, Texas A&M University, 1999
  - Sventlana Lazebnik, UNC, 2010
  - Noah Snavely, Cornell University, 2012
  - Xin Li, WVU, 2014
  - George Wolberg, City College of New York, 2015
  - Yao Wang and Zhu Liu, NYU-Poly, 2015

