Sample Problems For Exam 1

Fall 1998

This collection of questions is intended to give you an idea of different types of question that might be asked on the exam. This is not intended to represent an actual exam.
These questions cover chapters 1, 2 and 3 in the Applied Finite Mathematics, 5^{th} edition by S. T. Tan.

1. The distance from $(3, 7)$ and $(x, -1)$ is 10. Find x.
2. Find the equation of the line with x-intercept of 5 and y-intercept of 7.
3. Find the equation of the line through the point $(7, 2)$ and parallel to $4x + 2y = 7$.
4. Find the equation of the line through the point $(7, 2)$ and perpendicular to $4x + 2y = 7$.
5. Find the equation of the line through the point$(7, 2)$ and parallel to $y = 5$.
6. As a person descends into the ocean, pressure increases linearly. The pressure is 15 pounds per square inch on the surface and 30 pounds per square inch 33 feet below the surface. If y is the pressure in pounds per square inch and x is the depth below the surface in feet, write an equation that expresses the pressure in terms of the depth.
7. Auto-time, a manufacturer of 24-variable timers, has a monthly fixed cost of \$48,000 and a production cost of \$8 for each timer manufactured.
   (a) Find the Cost function.
   (b) What is the selling price of the timers, if the company has a profit of \$112,000 when selling 5,000 timers.
   (c) Find the Revenue function.
   (d) Find the Profit function.
   (e) How many items should be sold to break even?
8. The supply function for a product is given by $3x - 11p + 45 = 0$ and the demand function for this product is given by $2x + 7p - 56 = 0$. What is the equilibrium price and equilibrium quanity?
9. For what value of k does the system below have no solution?

   $$\begin{array}{rrr} 2x + & y = & 6 \\ x + & ky = & 1 \end{array}$$

10. The following data are representative of information in *Energy Policy*, March 1983. The data represents carbon dioxide (CO_2) emissions from coal-fired boilers (in units of 1000 tons) over a period of years between 1965 and 1977. For this problem let the time start with zero in 1965.(i.e. 1965 is zero)

Year(x)	0	5	8	9	10	11	12
CO_2 emission (y)	910	680	520	450	370	380	340

   (a) Determine the equation of the least-squares line for this data.
   (b) Sketch a scatter diagram and the least-squares line for the data.
   (c) Predict the carbon dioxide emissions in the year 1972.
   (d) In what year would we expect to have carbon dioxide emissions of 200?
   (e) Predict the carbon dioxide emissions in the year 1975.

11. Given an example of a matrice in row reduced form that describes a system with an infinite number of solutions.

12. Give an example of a matrix in reduce row echelon form with exactly one solution.

13. Give an example of a matrix in reduce row echelon form with no solution.

14. For the next two word problems do the following.
I) Define the variables that are used in setting up the system of equations.
II) Set up the system of equations that represent this problem.
III) Solve for the solution.
IV) If the solutions is parametric, then tell what restrictions can be placed on the parameter(s). Also give three specific solutions.

   (a) The management of a private investment club has a fund of \$300,000 earmarked for investment in stocks. To arrive at an acceptable overall level of risk, the stocks that management is considering have been classified into three categories: high-risk, medium-risk, and low-risk. Management estimates that high-risk stocks will have a rate of return of 16 percent per year; medium-risk stocks, 10 percent per year; and low-risk stocks, 4 percent per year. The investment in medium-risk stocks is to be twice the investment in stocks of the other two categories combined. If the investment goal is to have an average rate of return of 11 percent per year on the total investment, determine how much the club should invest in each type of stock.

   (b) A chemical manufacturer wants to purchase a fleet of 24 railroad tank cars with a combined carrying capacity of 250,000 gallons. Tank cars with three different carrying capacities are available: 6,000 gallons, 8,000 gallons, and 18,000 gallons. How many of each type of tank car should be purchased?

15. Solve the system of equations.

$$\begin{aligned} -2x + 2y + 4z &= 9 \\ 5x + y + 3z &= 17 \\ x + 3y + 5z &= 11 \\ -5x + 7y + 11z &= 14 \\ -9x + 41y + 69z &= 132 \end{aligned}$$

16. Solve for the variables x, y, z, and u. If this is not possible, then explain why

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} -2x & 0 \\ 3 & 4 \\ x+2 & 3 \end{bmatrix} - 3 \begin{bmatrix} y-1 & 2 \\ 1 & 2 \\ 4 & 2z+1 \end{bmatrix} = \begin{bmatrix} -7 & -2u \\ 0 & -2 \\ 8 & 10 \end{bmatrix}$$

17. Find the matrix J that makes the following true. If this is not possible, then explain why.

$$\begin{bmatrix} 0 & 8 & 1 \\ 7 & -6 & 0 \\ 3 & 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 5 & 2 & 0 \\ 6 & 6 & 1 \end{bmatrix} J = \begin{bmatrix} 7 & 0 & 6 \\ 0 & 1 & 4 \\ 3 & 7 & 0 \end{bmatrix}$$

18. Find a matrix A and a matrix B such that AB can be computed but BA can not be computed.

19. Use the following matrices for this problem. Compute the following operations. If it is not possible, then explain why.

$$A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \\ 0 & 2 \\ 4 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & -2 & 3 \end{bmatrix}$$

$D + C =$ $\qquad$ $D - 3B =$ $\qquad$ $DC =$

$DA =$ $\qquad$ $B + C^T =$ $\qquad$ $B^{-1} =$

$A^{-1} =$ $\qquad$ $E^{-1} =$

20. Problem # 41 section 2.5

21. Solve the systems of equations by using inverses.

$$\begin{aligned} 2x + y + z &= b_1 \\ 5x + 2y + z &= b_2 \\ 3x + 2y + 4z &= b_3 \end{aligned}$$

    (a) $b_1 = 2$, $b_2 = -1$, $b_3 = 0$

    (b) $b_1 = 3$, $b_2 = 4$, $b_3 = -2$

22. Graph the following set of inequalities; label all corner points; shade the feasible region; and tell if the feasible region is bounded.

$$\begin{array}{rcrcr} -x & + & y & \leq & 0 \\ x & + & y & \leq & 20 \\ 8x & + & 30y & \leq & 240 \end{array}$$

23. **SET UP ONLY:** Set up the constraints and the objective function for this word problem. Be sure to label the variables.

    The Soundex Company produces two models of clock radios. Model A requires 15 minutes of work on assembly line I and 10 minutes of work on assembly line II. Model B requires 10 minutes of work on assembly line I and 12 minutes of work on assembly line II. At most, 25 hours of assembly time on Line I and 22 hours of assembly time on Line II are available per day. It is anticipated that Soundex will realize a profit of $12 on model A and $10 on Model B. How many clock radios of each model should be produced per day in order to maximize Soundex's profit?

24. The following system of inequalities are constraints in a linear programming problem. Graph the feasible region. Label all lines and corner points.

$$\begin{aligned} x - y &\geq -5 \\ x + y &\leq 10 \\ 10x + 9y &\geq 45 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

25. Use the feasible region from problem 24. At what point is the objective function $f = x + 3y$ maximized and what is the maximum value? (if not possible, explain why.)

26. Use the feasible region from problem 24. At what point is the objective function $f = x + 3y$ minimized and what is the minimum value? (if not possible, explain why.)