Math 141

Test II (Ch. 3, 4, & 6)

Wednesday, October 18, 1995

Name ________________________________

ID# ___________________________

Seat Number _________

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-- You must show all appropriate work to receive full credit.

-- There are 100 points possible.  Point values for each problem are as indicated.

-- You will have 50 minutes to complete this exam.  Budget your time accordingly.

-- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

-- If you need more space to work a problem, do so on the back of the page and indicate this on the front of the page.

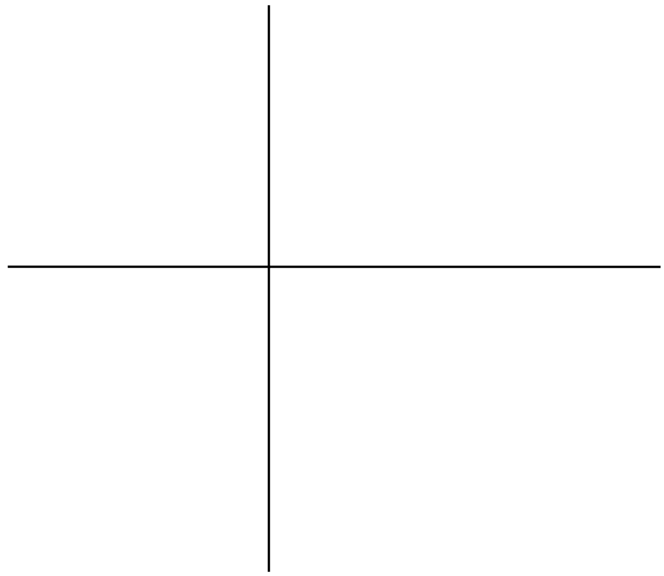
**Good Luck!**

**Problem 1** -- (15 points) Formulate, BUT **DO NOT SOLVE**, the following standard maximization problem. Define your variables, write out the objective and constraints, and set up the initial simplex tableau.

The Math Club is planning a fund-raising cookie sale. They plan to bake sugar cookies, peanut butter cookies, and pecan sandies in batches of 3 dozen. Each batch of sugar cookies requires 2 eggs, 3 cups of flour, and $1\frac{1}{8}$ cups of sugar. Batches of peanut butter cookies use 1 egg, $1\frac{1}{4}$ cups of flour, and 1 cup of sugar; whereas pecan sandies use 2 cups of flour, $\frac{1}{3}$ cup of sugar and no eggs. Ten dozen eggs, 300 cups of flour, and 200 cups of sugar have been donated to the club, and all other necessary ingredients are on hand in sufficient quantities. If a dozen sugar cookies sells for $1.50, a dozen peanut butter cookies sells for $1.75, and a dozen pecan sandies sells for $2.25, how many batches should they bake in order to maximize their income? (Watch your units!!)

**Problem 2** -- (10 points) Find the solution set of the following system of inequalities by graphing them. State whether the solution set is bounded or unbounded.

$$x+y\geq 4$$
$$-x+y\geq 0$$
$$0\leq y\leq 8$$
$$x\geq 0$$



**Problem 3** -- (15 points)
Solve the following linear programming problem graphically.

Minimize $C=2x+4y$
Subject to
$$x+y\leq 20$$
$$-x+y\leq 10$$
$$x+2y\geq 20$$
$$0\leq x\leq 10$$
$$y\geq 0$$

**Problem 4** -- (5 points each) For each of the following simplex tableaus, state whether a solution has been reached. If so, give the solution for all the variables indicated. If a solution has not been reached, then circle the next pivot element and write the row operations that should be performed in the next step.
**(Do not carry out the row operations!)**

| x | y | z | $s_1$ | $s_2$ | $s_3$ | P | |
|---|---|---|---|---|---|---|---|
| 4 | 2 | 0 | 1 | $-\frac{1}{2}$ | 0 | 0 | 2 |
| -1 | -2 | 1 | 0 | $\frac{1}{2}$ | 0 | 0 | 1 |
| 3 | 1 | 0 | 0 | -1 | 1 | 0 | 0 |
| -250 | -300 | 0 | 0 | 400 | 0 | 1 | 400 |

| x | y | $s_1$ | $s_2$ | $s_3$ | P | |
|---|---|---|---|---|---|---|
| 1 | 0 | -1 | 0 | $-\frac{1}{2}$ | 0 | 4 |
| 0 | 0 | $\frac{1}{6}$ | 1 | $\frac{1}{2}$ | 0 | $\frac{5}{6}$ |
| 0 | 1 | 1 | 0 | 1 | 0 | 3 |
| 0 | 0 | 300 | 0 | 200 | 1 | 500 |

| x | y | z | u | v | w | P | |
|---|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | 0 | $\frac{1}{4}$ | 1 | $-\frac{1}{4}$ | 0 | 0 | $\frac{19}{2}$ |
| $\frac{1}{2}$ | 1 | $\frac{3}{4}$ | 0 | $\frac{1}{4}$ | 0 | 0 | $\frac{21}{2}$ |
| 1 | 0 | 3 | 0 | 0 | 1 | 0 | 15 |
| -1 | 0 | $-\frac{1}{2}$ | 0 | $\frac{3}{2}$ | 0 | 1 | 63 |

**Problem 5** -- (6 points) The Byron Hopkins pen company makes 2 types of ballpoints pens: a silver model and a gold model. The silver model requires 1 minute in the grinder, 3 minutes in the bonder, and 2 minutes in the packager. The gold model requires 3 minutes in the grinder, 4 minutes in the bonder, and 4 minutes in the packager. Because of maintenance procedures, the grinder can only be operated no more that 30 hours per week, the bonder no more than 50 hours per week, and the packager no more than 40 hours per week. The company makes $5 profit on the silver model and $6 on each gold model sold. How many of each pen type should be produced and sold to maximize the company’s profit?

If ***x*** represents the number of the silver model pens produced and sold and ***y*** represents the number of gold models pens produced and sold, then this standard maximization problem is written as follows:

Maximize $P=5x+6y$

Subject to $x\geq 0,\ \ y\geq 0$

$x+y\leq 1800$ (minutes of grinder time)

$3x+4y\leq 3000$ (minutes of bonder time)

$2x+4y\leq 2400$ (minutes of packager time)

We set this up for the simplex method:

$$x+y+s_1=1800 \quad \text{(minutes of grinder time)}$$
$$3x+4y+s_2=3000 \quad \text{(minutes of bonder time)}$$
$$2x+4y+s_3=2400 \quad \text{(minutes of packager time)}$$
$$-5x-6y+P=0$$

Putting these equations into a simplex tableau and pivoting a few times we have the following final tableau:

| $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $P$ | **Constant** |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | $\frac{1}{2}$ | $-\frac{5}{4}$ | 0 | 300 |
| 1 | 0 | 0 | 1 | -1 | 0 | 600 |
| 0 | 1 | 0 | $-\frac{1}{2}$ | $\frac{3}{4}$ | 0 | 300 |
| 0 | 0 | 0 | 2 | $\frac{3}{2}$ | 1 | 4,800 |

a) (4 points) Write the solution to this problem.

b) (2 points) Are there any left over resources? If so, list any left over resources and the amount that is left.

**Problem 6** -- Let $U$ = { all employees at Scott & White }
$A$ = { administrators }
$D$ = { doctors }
$N$ = { nurses }
$M$ = { male employees }

a) (3 points each) Describe each of the following sets.

i) $M^c \cap N =$

ii) $(D \cup A) \cap M =$

b) (3 points each) Write the set that represents each of the given statements.

i) The set of all doctors or nurses that are not administrators =

ii) The set of all female doctors that are administrators =

**Problem 7** -- (3 points) How many distinct permutations are there of the letters (ignore any spaces and punctuation) in the following phrase:

*BEAT THE HELL OUTTA BAYLOR*!

**Problem 8** -- (3 points) Ordinary Texas license plates are formed by 3 letters, followed by 2 digits, followed by one more letter. How many possible license plates are there if the last letter cannot be a vowel?

**Problem 9** -- (3 points each) Assume that you have a standard 52-card deck of playing cards.

a) How many 5-card poker hands are there that consist of exactly three aces and two face cards?

b) How many 5-card poker hands are there that have <u>at least</u> 3 aces?

**Problem 10** -- (15 points) A psychologist took 20 monkeys and spent a week trying to teach them to ride a tricycle, another week teaching them to do chin-ups, and another week teaching them to shoot a basketball into a low hoop. The results of his experiment were

2 monkeys could be taught to do all three activities
2 monkeys could be taught to ride a tricycle and shoot a basketball only
1 monkey could be taught to do chin-ups and shoot a basketball only
3 monkeys could be taught to do ride a tricycle and do chin-ups only
9 monkeys total could be taught to ride a tricycle
8 monkeys total could be taught to shoot a basketball
5 monkeys could not be taught to do any of the activities

<u>Draw a Venn diagram</u> and answer the following questions:

How many monkeys were taught to

a) do chin-ups but not ride a tricycle?

b) do at least two of the activities?

c) do exactly two of the activities?