Random Problems 3_1 to 3_3
for Finite Math
Solutions and Hints

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Problems based on sections from:
<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

<u>Section 3.1, 3.2 and 3.3:</u>

<u>Problem 1</u>:
Find the minimum and maximum for P = 5x + 2y subject to
$2x + 4y \geq 16$
$-x + 3y \geq 7$
$x \geq 0$ and $y \geq 0$
Using the method of corners:

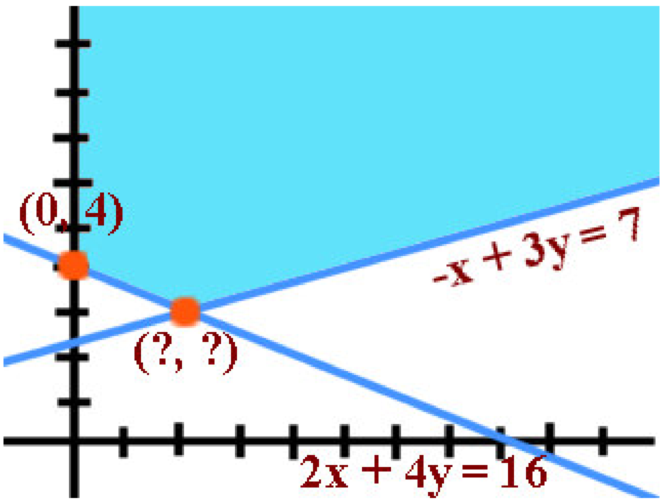
<u>For the line 2x + 4y = 16:</u>
Let x = 0 → y = 4, so one point is (0, 4)
Let y = 0 → x = 8, so another point is (8, 0)
Draw a line through the points (0, 4) and (8, 0)
Test the point (0,0) in the Equation $2x + 4y \geq 16$:
$3x + 4y \geq 16$ → $0 + 0 \geq 16$ which is false
So the false side is below the line.

<u>For the line –x + 3y = 7:</u>
Let x = 0 → y = 7/3, so one point is (0, 7/3)
Let y = 3 → x = 2, so another point is (2, 3)
Draw a line through the points (0, 7/3) and (2, 3)
Test the point (0, 0) in the Equation $-x + 3y \geq 7$
$-x + 3y \geq 7$ → $0 + 0 \geq 7$ which is false
So the false side of the line is below the line

For the line $x \geq 0$:
The false side of this should obviously be everything left of the y-axis.

For the line $y \geq 0$:
The false side of this line should obviously be everything below the x-axis.

Your graph should look something like this:
NOTE: I have shaded the TRUE side.



Now you must find the (?, ?) point which is the intersection of the two lines.
So solve the system of equations:

Eq 1: $2x + 4y = 16$
Eq 2: $-x + 3y = 7$

Solve Eq 2 for y:

$-x + 3y = 7 \rightarrow 3y = 7 + x \rightarrow y = 7/3 + (1/3)x$

Substitute $7/3 + (1/3)x$ in for y into Eq 1:

$2x + 4y = 16 \rightarrow 2x + 4*(7/3 + (1/3)x) = 16$
$\rightarrow 2x + 21/3 + (4/3)x = 16$
$\rightarrow (10/3)x = 9$
$\rightarrow x = 27/10 = 2.7$

And put 2.7 in for x into Eq 2 and solve for y:

$-x + 3y = 7 \rightarrow -2.7 + 3y = 7$
$\rightarrow y = 97/30$

Now put your corner points into your objective function:

(0, 4) : $5x + 2y = 5*0 + 2*4 = 8$
(27/10, 97/30) : $5x + 2y = 5*(27/10) + 2*(97/30) = 599/30 \approx 19.96666$

So (0, 4) would appear to yield the minimum value of 8.

However 19.9666 is NOT a maximum for consider that the top side of the graph is unbounded, so we can put 0 for x and 100 for y and get 5x + 2y = 0 + 200 = 200. And clearly 200 > 19.6666.

So since the graph is unbounded at the top we say there is no maximum.

You may also want to consider what happens when we do not have the constraints as ≤ or ≥ but just < or just >. Can we use the corners of intersection then? You may want to ask your instructor about this.

<u>Problem 2</u>:

Minimize $P = 2x + 3y$ subject to

$5x + y \geq 20$

$x + y \geq 12$

$x + 3y \geq 18$

$x \geq 0,\ y \geq 0$

Using the Method of Corners.

<u>For the line $5x + y = 20$:</u>

Let $x = 0$ and solve for y gives $y = 20$. So one point is (0, 20)
Let $y = 0$ and solve for x gives $x = 4$. So another point is (4, 0)
So draw the line through the points.
Test the point (0, 0) in the inequality: $5x + y \geq 20$
Gives $5*0 + 0 \geq 20 \rightarrow 0 \geq 20$ which is false
So the false side is below the line.

<u>For the line $x + y = 12$:</u>

Let $x = 0$ gives $y = 12$. So one point is (0, 12)
Let $y = 0$ gives $x = 12$. So another point is (12, 0)
So draw the line through the points.
Test the point (0, 0) in the inequality $x + y \geq 12$
Gives $0 + 0 \geq 12 \rightarrow 0 \geq 12$ which is false.
So the false side is below the line.

<u>For the line $x + 3y = 18$:</u>

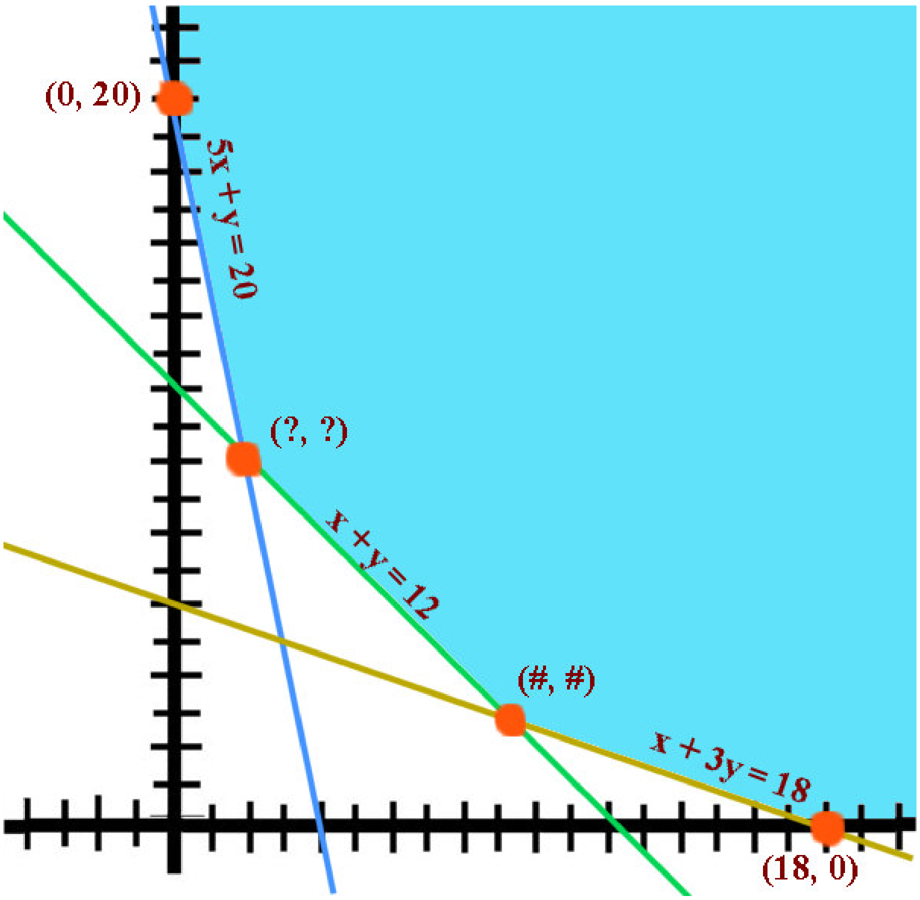
Let $x = 0$ gives $y = 6$. So one point is (0, 6)
Let $y = 0$ gives $x = 18$. So another point is (18, 0)
Draw the line through the points.

Test the point (0, 0) in the inequality $x + 3y \geq 18$

Gives) + 3*0 ≥ 18 → 0 ≥ 18 which is false

So the false side is below the line.

Your graph should look like the below (note the TRUE side is shaded):



To find the point (?, ?) you need to solve the system:

Eq 1: $5x + y = 20$

Eq 2: $x + y = 12$

So solve Eq 2 for y → $y = 12 – x$.

Sub 12 – x in for y into Eq 1:

$5x + y = 20 \rightarrow 5x + (12 – x) = 20$

$\rightarrow 4x = 8$

$\rightarrow x = 2$

Put x = 2 into Eq 2:

$x + y = 12 \rightarrow 2 + y = 12 \rightarrow y = 10$

So (?, ?) is really (2, 10)

To find the point (#, #) you need to solve the system:

Eq 1: $x + 3y = 18$
Eq 2: $x + y = 12$

So solve Eq 2 for y → $y = 12 – x$.
Sub $12 – x$ in for y into Eq 1:

$x + 3y = 18$ → $x + 3*(12 – x) = 18$
→ $x + 36 – 3x = 18$
→ $-2x = -18$
→ $x = 9$

Sub 9 in for x back into Eq 2:

$x + y = 12$ → $9 + y = 12$ → $y = 3$

So the point (#, #) is really (9, 3)

Now to find our minimum of $P = 2x + 3y$ we put in each corner point and calculate the value of P, the one that gives us the minimum value of P gives us our answer.

(0, 20) → $2x + 3y = 60$
(2, 10) → $2x + 3y = 4 + 20 = 34$
(9, 3) → $2x + 3y = 18 + 9 = 27$
(18, 0) → $2x + 3y = 36$

So the minimum value of 27 occurs at x = 9 and y = 3.

<u>Problem 3</u>:
Using the Method of Corners set up and solve the following linear programming problem:
A farmer raises only chickens and pigs. Each chicken produces \$6 in profit whereas each pig is worth \$20 in profit. The farmer wants to raise at most 16 animals. Further the farmer wishes to have at most 10 chickens. To raise each chicken costs the farmer \$5 and to raise each pig costs \$15. The farmer is restricted to spend at most \$180. How many of each animal should he raise if he wishes to maximize his profits?

Let x = number of chickens raised
Let y = number of pigs raised

The objective function is profit, which we are to maximize.
The profit from each chicken is \$6.
The profit from each pig is \$20.

So the total profit is: $P = 6x + 20y$

The total number of animals to be raised is at most 16, so our first constraint is:

Con 1: $x + y \leq 16$

The farmer also wants at most 10 chickens, which gives the second constraint:

Con 2: $x \leq 10$

And the farmer has only $180 to spend.
Each chicken costs $5 to raise.
Each pig costs $15 to raise.
So we get our third constraint:

Con 3: $5x + 15y \leq 180$

And of course we cannot raise negative chickens or pigs so we get constraints 4 and 5:

Con 4: $x \geq 0$
Con 5: $y \geq 0$

So our total system is:
Maximize $P = 6x + 20y$ subject to
$x + y \leq 16$
$x \leq 10$
$5x + 15y \leq 180$
$x \geq 0, y \geq 0$

Now we need to graph it and find the corners so we can find the maximal solution.

<u>For the line $x + y = 16$:</u>

Let $x = 0$ and solve for y → $y = 16$, so one point is (0, 16)
Let $y = 0$ and solve for x → $x = 16$, so another point is (16, 0)
Draw the line through the points.
Test the inequality $x + y \leq 16$ with the point (0, 0):

$0 + 0 \leq 16 \rightarrow 0 \leq 16$ is true

So the true side is below the line.

<u>For the line $5x + 15y = 180$:</u>

Let $x = 0$ and solve for y → $y = 12$, so one point is (0, 12)
Let $y = 0$ and solve for x → $x = 36$, so another point is (36, 0)
Draw the line through the points.
Test the inequality $5x + 15y \leq 180$ with the point (0, 0)

$0 + 0 \leq 180 \rightarrow 0 \leq 180$ is true

So the true side is below the line.

<u>For $x \leq 10$:</u>

Clearly the true side of this is everything left of the vertical line $x = 10$.

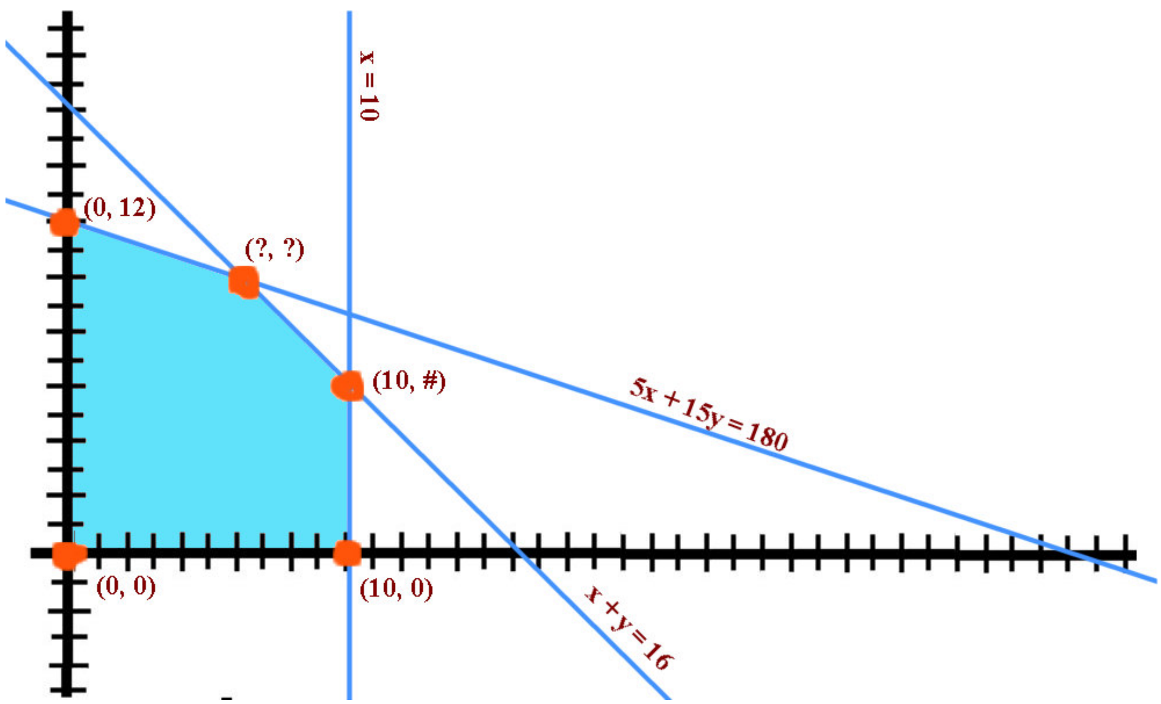
For $x \geq 0$:

Clearly the true side is everything to the right of the y-axis ($x = 0$).

For $y \geq 0$:

This is obviously everything above the x-axis ($y = 0$).

So your graph should look something like:
(Note the below has the TRUE portion shaded)



And now we need to find the point (?, ?) where $5x + 15y = 180$ intersects $x + y = 16$.
So solve the system:

Eq 1: $5x + 15y = 180$

Eq 2: $x + y = 16$

Solve Eq 2 for y → $x = 16 - y$
Substitute that in for x into Eq. 1:

$5x + 15y = 180$ → $5*(16 - y) + 15y = 180$

→ $80 - 5y + 15y = 180$

→ $10y = 100$

→ $y = 10$

Put 10 in for y into Eq.2 and solve for x:

$x + y = 16$ → $x + 10 = 16$ → $x = 6$

So (?, ?) is really the point **(6, 10)**.

Next we must find out what point (10, #) really is,
or rather where x = 0 intersects x + y = 16.

So solve the system:

Eq 1: $x + y = 16$

Eq 2: $x = 10$

Put 10 in for x into equation 1 and solve for y:

$x + y = 16 \rightarrow 10 + y = 16 \rightarrow y = 6$

So (10, #) is really **(10, 6)**.

Now we have all our corner points.
So we put them into our objective function: $P = 6x + 20y$
And select the point which produces the largest result (as we are trying to maximize P).

(x, y) = point	**P = value of 6x + 20y**
(0, 0)	0
(0, 12)	*0+20*12* = 240
(6, 10)	*6*6 + 20*10* = 236
(10, 6)	*6*10+20*6* = 180
(10, 0)	*6*10 + 0*20* = 60

So the maximum profit is 240 at point (0, 12)

Thus we conclude:

The farmer will achieve a maximum profit of $240 if he raises no chickens and 12 pigs.