MATH 141 – From Students Assignment 1

Spring and Fall 2002

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Problem 1:

Find the equation of the line parallel to $y = 6x - 9$ through the point (2, 4).

Solution:

Since we want a line parallel to the one given we first
find the slope of the line given by $y = 6x - 9$.
Notice that equation is in slope-intercept form (i.e. $y = mx + b$)
So we easily see that $m = 6$ = slope of the line.

So now we know our desired line has slope $m = 6$ and goes through point (2, 4).
All we need to do is plug stuff into the point-slope form (see page 20 of your book):

$y - y_1 = m(x - x_1) \rightarrow \quad y - 4 = 6*(x - 2)$

And we probably want that in slope-intercept form (i.e. $y = mx + b$) so we simplify a little:

$y - 4 = 6*(x - 2) \rightarrow \quad y - 4 = 6x - 12$

$$\mathbf{y = 6x - 8}$$

Problem 2:

It has been determined that C.M. Wholesale Flowers will charge \$3 per flower when 100 flowers are ordered and \$1.57 per flower when 500 flowers are ordered.
Find the linear equation which represents the sale of flowers.
What will they charge for a 250 flower order?

Solution:

This is a demand equation problem. See your book page 34.

Let p denote the price per flower.
Let x denote the number (quantity) of flowers being sold.

We are given when $p = \$3$, $x = 100$ and when $p = \$1.57$, $x = 500$.

So effectively we are given 2 points and we need to find a line through them.
Our two points are:
(100, \$3) and (500, \$1.57)

So the slope of our line is:

$$m = \frac{p_1 - p_2}{x_1 - x_2} = \frac{3 - 1.57}{100 - 500} = -\frac{1.43}{400} = -0.003575$$

We now go back to our point-slope form of a line using (100, \$3) for our point and –0.003575 as our slope to get the following:

$y - y_1 = m(x - x_1) \rightarrow \; y - 3 = -0.003575*(x - 100)$

And we simplify into slope-intercept form (i.e. y = mx + b):
$y - 3 = -0.003575*(x - 100) \rightarrow \quad y - 3 = -0.003575x + 0.3575$
$y = -0.003575x + 3.3575$

Notice if we put 500 in for x we get y = 1.57:
$y = -0.003575*500 + 3.3575 = -1.7875 + 3.3575 = 1.57$

So the demand equation is: **y = -0.003575x + 3.3575**

And we are also asked what the price of 250 flowers would be, so we put 250 in for x and get:
$y = -0.003575*250 + 3.3575 = -0.89375 + 3.3575 = 2.46375$

While math say we round that down to a cost of \$2.46, most businesses would round up and **charge \$2.47 per flower if 250 flowers were ordered**. Most likely either answer will be accepted – but you might ask your professor.

<u>Problem 3</u>:

The following table represents the number of households (in millions) who use online banking each year.

year:	1995	1996	1997	1998	1999	2000
households:	4.5	7.5	10	13	15.6	18

This a least squares problem – you probably have a program for the calculator available to solve this type of problem. I STRONGLY encourage you to figure out how to use the program as it makes this extremely easy.

a. Find the best fit line.

Let x = 1 denote the year 1995

	x	y	x^2	xy
	1	4.5	1	4.5
	2	7.5	4	15
	3	10	9	30
	4	13	16	52
	5	15.6	25	78
	6	18	36	108
Sum	21	**68.6**	**91**	**287.5**

So from the table we create 2 equations with 2 unknowns: m and b, which will in the end represent the slope and y-intercept of our desired best fit line.
Notice the highlighting on the numbers below and in the table – this should reflect where the numbers come from.
Notice also we have 6 pairs of data.

Eq 1: 6*b + 21m = **68.6**
Eq 2: 21*b + **91**m = **287.5**

We now solve the system of equations.

We will first solve equation 1 for b:

6b + 21m = 68.6 → 6b = 68.6 – 21m

b = (68.6 – 21m) / 6

$$b = \frac{343}{30} - \frac{7}{2}m$$

And we substitute that value for b into equation 2 and solve for m:

21*b + 91m = 287.5 →

$$21*(\frac{343}{30} - \frac{7}{2}m) + 91m = 287.5$$

$$\frac{2401}{10} - \frac{147}{2}m + 91m = 287.5$$

240.1 + 17.5*m = 287.5

17.5*m = 47.4

m = 2.70857

We then take that value of m and sub it back into Equation 1 to find b:

6b + 21m = 68.6 → 6b + 21*(2.70857) = 68.6

6b + 56.88 = 68.6

6b = 11.72

$$b = \frac{293}{150} \approx 1.95333$$

So the slope of the best fit line is m = 2.70857

and the y-intercept of the best fit line is $b = \frac{293}{150}$

So using the slope-intercept form of a line (i.e. y = mx + b) we find that the equation of the best fit line is:

y = 2.70857*x + (293 / 150)

b. Estimate the number of families online by 2010

Since we used x = 1 for 1995 consider:
1995 → 1
1996 → 2
1997 → 3
1998 → 4
1999 → 5
2000 → 6
2001 → 7
2002 → 8
2003 → 9
2004 → 10
2005 → 11
2006 → 12
2007 → 13
2008 → 14
2009 → 15
2010 → 16

So we put 16 into the equation found in part a:

$$y = 2.70857*x + (293 / 150) = 2.70857*16 + (293 / 150)$$
$$\approx 45.2904533333$$

Recall y is an estimate in MILLIONS so move the decimal right 6 places rounding to the nearest full household and you get the answer:

45,290,453 households

c. Predict the year 25,000,000 households will use online banking

Again we will use the answer from part a.
But this time we will put 25 in for y and solve for x:

$y = 2.70857*x + (293 / 150) \rightarrow$

$25 = 2.70857*x + (293 / 150)$
$25 – (293 / 150) = 2.70857*x$
$(3457 / 150) = 2.70857*x$
$8.508794924 \approx x$

Recall again x = 1 represented the year 1995,
looking at the table in part b we see that x = 8 represents the year 2002…
So **about half way through 2002** (i.e. around now) 25 million households are using online banking.

Problem 4:

A movie theater charges $8.00 per person to go to the movies. There is a fixed cost of $800 and the theater has an additional cost of $2.75 per ticket sold.

a. Find the cost equation.
This is straight forward:
Cost = Fixed cost + (cost per ticket) * (number of tickets)

C = 800 + 2.75*x

b. Find the revenue equation.
Revenue is how much the theater makes total.

Revenue = (price per ticket) * (number of tickets)

R = 8.00 * x

c. Find the profit equation.
Profit = Revenue – Cost
P = R – C   (sub in answers for parts a and b for R and C)

P = (8.00*x) – (800 + 2.75*x)   now simplify
P = 8x – 800 + 2.75x

P = 5.25x – 800

Problem 5:

Given the points (6, -2) and (-8, 3)

These problems are similar to ones worked out above – just phrased differently.

a. Find the line through the points.

First find the slope of the desired line (recall rise over run)

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2-3}{6-(-8)} = \frac{-5}{14}$$

And use point-slope form to find the line:

$y - y_1 = m*(x - x_1) \rightarrow y - (-2) = (-5/14)*(x - 6)$ *now simplify to slope-intercept form*

$y + 2 = (-5/14)x + (30/14)$

$y + 2 = (-5/14)x + (15/7)$

$$\mathbf{y = (-5 / 14)x + (1 / 7)}$$

b. Is the point (0, 1) on the line?

For this all we need to do is sub in 0 for x into our answer for part (a).
If we find that y = 1 then the point (0, 1) is on the line, otherwise it is not.

$y = (-5/14)x + (1/7) \rightarrow (-5/14)*0 + (1/7) = 1 / 7$.

Since $1/7 \neq 1$, clearly **the point (0, 1) is NOT on the line.**

c. Find the parallel line through the point (10, -2)

For this all we need to do is not from part (a) our line has a slope = m = (-5/14).
We then use the point-slope form to create the new equation
– using (10, -2) as our (x_1, y_1):

$y - y_1 = m*(x - x_1) \rightarrow y - (-2) = (-5/14)*(x - 10)$ *now simplify to slope-intercept form*

$y + 2 = (-5/14)x + (50/14)$

$y + 2 = (-5/14)x + (25 / 7)$

$$\mathbf{y = (-5/14)x + (11 / 7)}$$

d. Find the perpendicular line through the point (-4, 5)

This is the same process as part (c) only we use the negative reciprocal of –5/14 because this line is perpendicular to the one in part (a).

The negative reciprocal of (-5/14) is of course 14/5.
We now use the point-slope form to create the new equation – using (-4, 5) as our (x_1, y_1):

$$y - y_1 = m*(x - x_1) \rightarrow y - 5 = (5/14)*(x - (-4))$$
$$y - 5 = (5/14)*(x + 4)$$
$$y - 5 = (5/14)x + (20/14)$$
$$y - 5 = (5/14)x + (10 / 7)$$
$$\mathbf{y = (5 / 14)x + (45 / 7)}$$

Problem 6:

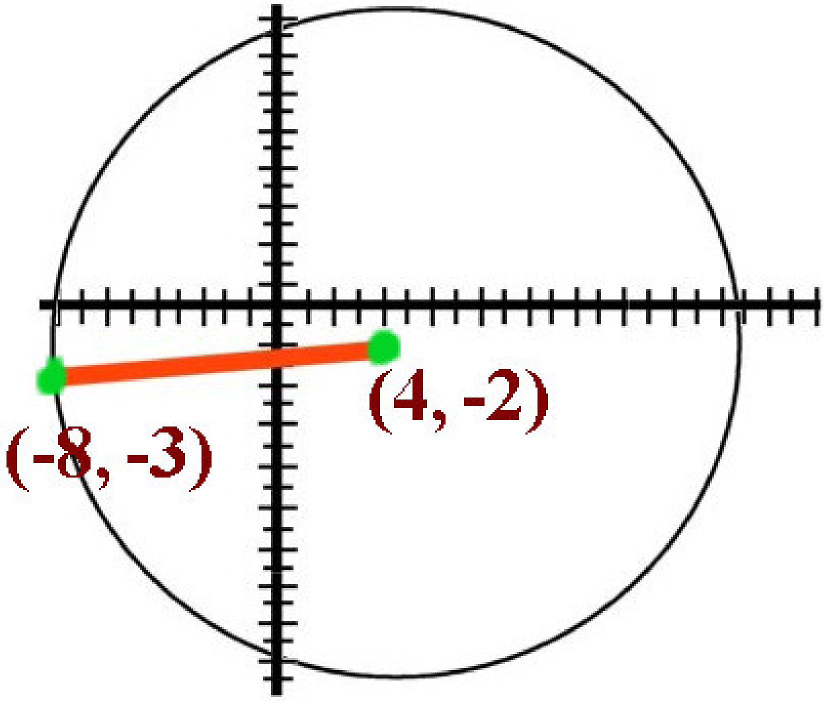
Find the equation of a circle with center (4, -2) through the point (-8, -3)

Solution:
Since we know the equation of a circle with center C(h, k) and radius r is given by:

$$(x - h)^2 + (y - k)^2 = r^2$$

and
we are given the center C(h, k) = (4, -2) all we need to do is find the SIZE of the radius.

Since we know the center is at (4, -2) and the circle goes through the point (-8, 3) we know the radius must be the distance between these points.
Picture:



So we first calculate the distance between the points to determine what the radius r is:

$r^2 = (h - x_1)^2 + (k - y_1)^2 \rightarrow$ $r^2 = (4 - -8)^2 + (-2 - -3)^2$

$r^2 = 12^2 + 1^2$

$r^2 = 144 + 1$

$\mathbf{r^2 = 145}$

$r = (145)^{1/2}$ (r = square root of 145)

So from our general equation we can now plug h, k and r^2 in:

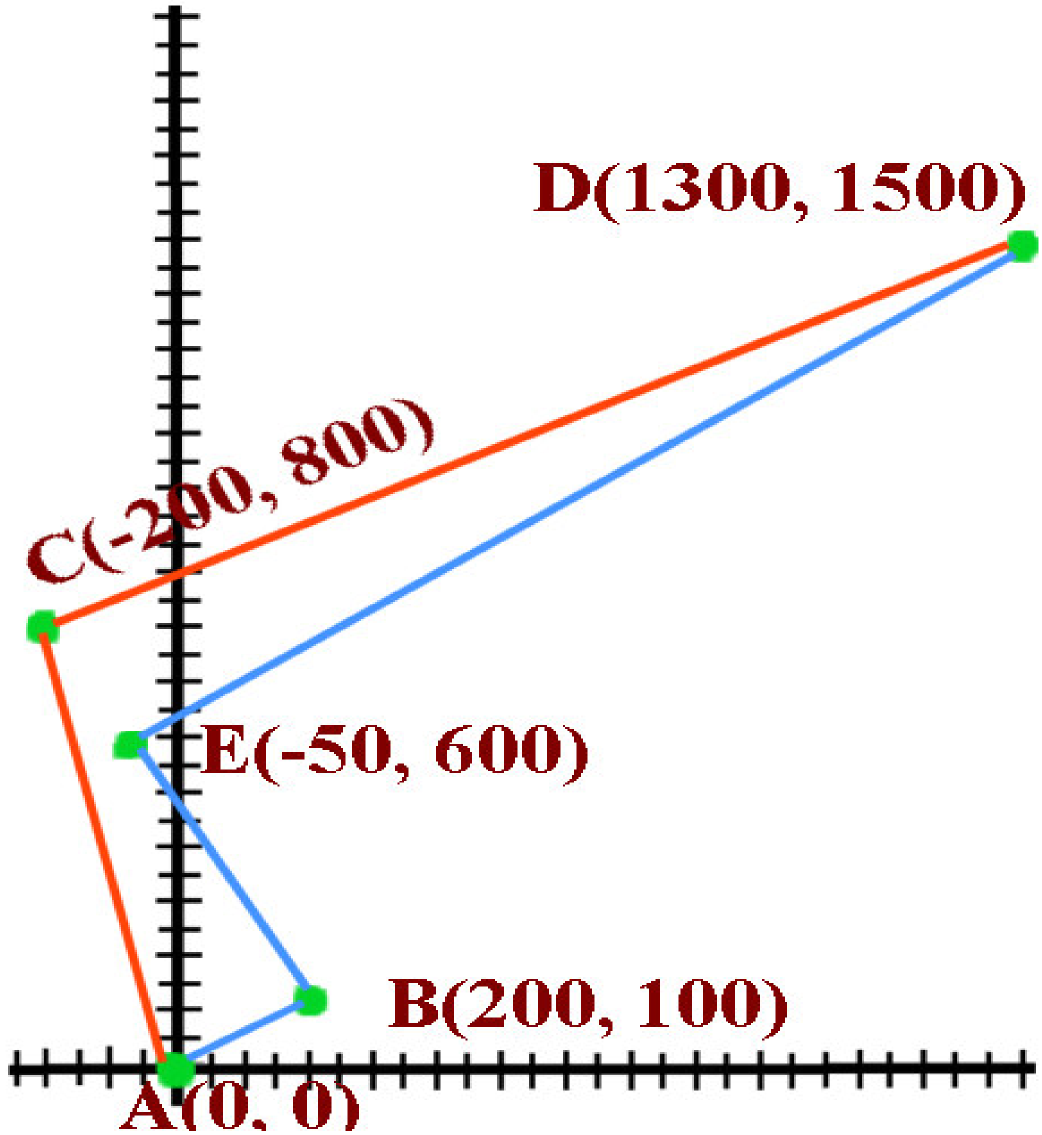
$(x - h)^2 + (y - k)^2 = r^2 \rightarrow$ $(x - 4)^2 + (y - -2)^2 = 145$

So simplifying we get the answer: $\mathbf{(x - 4)^2 + (y + 2)^2 = 145}$

Some professors may want you to expand that out.

Problem 7:

For this problem you are given 5 points and two highways both going from point A to point D. The below picture should summarize the information satisfactorily:



Notice we assumed the second route went first to B then to E then to D (as opposed to going to E then B then D). You may want to determine what the distance is both ways. Or at least ask your professor which way to assume.

Problem 8:

Find the equation of the line through the point (-0.2, 0.7) and perpendicular to $y = \frac{-6}{5}x + \frac{4}{5}$.

This is identical to problem 2, except we need our line to be perpendicular to the given line. We notice that the given line is again in slope-intercept form and see that its slope is

–6/5. Since we need our line perpendicular we will take the **negative reciprocal** of –6/5 and say our line has a slope of 5/6.

So now we know our desired line has slope m = 5/6 and goes through point (-0.2, 0.7). All we need to do is plug stuff into the point-slope form (see page 20 of your book):

$y - y_1 = m(x - x_1) \rightarrow$ $y - 0.7 = (5/6)*(x - -0.2)$
$y - 0.7 = (5/6)*(x + 0.2)$

And we probably want that in slope-intercept form (i.e. y = mx + b) so we simplify a little:

$y - 0.7 = (5/6)*(x + 0.2) \rightarrow$ $y - 0.7 = (5/6)x + 1/6$ *(recall 0.2 = 2/10, 5/6*2/10 = 10 / 60)*
$y - 7/10 = (5/6)x + 1/6$
$y = (5/6)x + 1/6 + 7/10$
$y = (5/6)x + 10/60 + 42/60$
$y = (5/6)x + 52/60$

$$\mathbf{y = (5/6)x + 13/15}$$
