MATH 141 – Homework 1 – Random Book

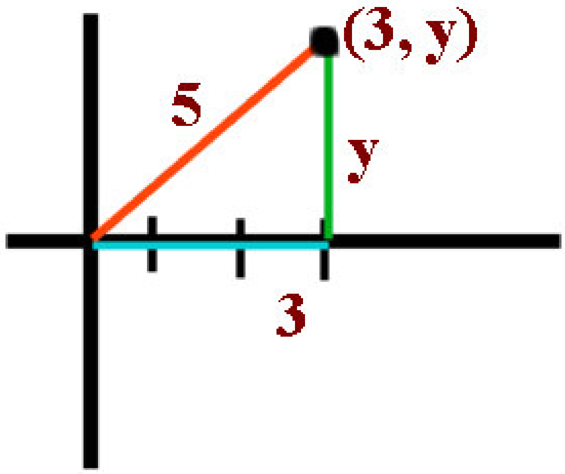
1.1: #26,
1.2: #46, 78,
1.3: #16, 38,
1.4: #22, 24,
1.5: #14

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!! This is designed to assist you in the event you get stuck. If you do not do the work you will NOT pass the tests.

Section 1.1, Problem 26:

Find the **coordinates** of the points that are 5 units away from the origin and have an x-coordinate equal to 3.

Picture:



So use the Pythagorean theorem $x^2 + y^2 = d^2$ with $x = 3$ and distance $d = 5$:

$3^2 + y^2 = 5^2 \rightarrow$ $9 + y^2 = 25$
$y^2 = 25 - 9$
$y^2 = 16$
$y = 4$ **or** -4

So the coordinates are: **(3, 4)** **<u>or</u>** **(3, -4)**

Section 1.2, Problem 46:

Find an equation of the line that passes through the point (2, 4) and is perpendicular to the line $3x + 4y - 22 = 0$.

First find the slope of the line: $3x + 4y - 22 = 0$
We will do this by putting it into slope-intercept form (i.e. $y = mx + b$)

$$3x + 4y - 22 = 0 \rightarrow \quad 3x + 4y = 22$$
$$4y = 22 - 3x$$
$$4y = -3x + 22$$
$$y = (-3x + 22) / 4$$
$$y = (-3/4)x + 22/4$$

So $m = -3/4$ and $b = 22/4$

So the slope of the line: $3x + 4y - 22$ is $-3/4$.
The line we must find must be perpendicular to that line so our desired line will have a slope equal to the negative reciprocal of $-3/4$ which is $4/3$.

So our desired line has slope = $m = 4/3$ and goes through the point (2, 4).
To find the equation of our desired line,
we will use point-slope form (i.e. $y - y_1 = m(x - x_1)$)

From what we know $x_1 = 2$, $y_1 = 4$ and $m = 4/3$.
So we will plug that in and see the equation of our desired line is:

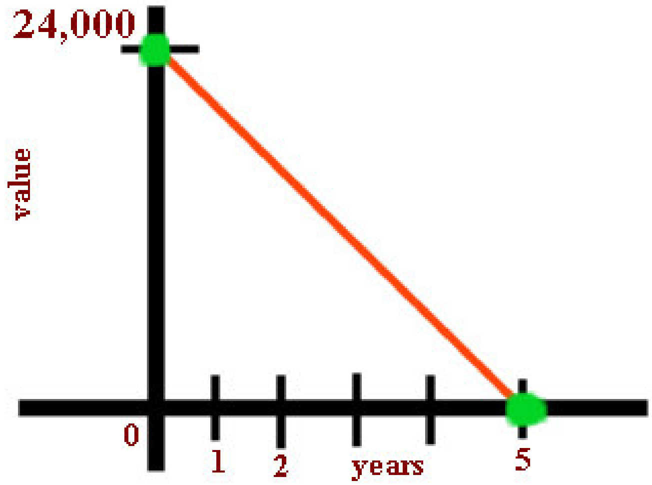
$$y - y_1 = m(x - x_1) \rightarrow \quad y - 4 = 4/3(x - 2) \qquad \text{(and we simplify to slope-intercept form)}$$
$$y - 4 = (4/3)x - 8/3$$
$$\mathbf{y = (4/3)x + 4/3}$$

Section 1.3, Problem 16:

An automobile purchased for use by the manager of a firm at a price of \$24,000 is to be deprecated using the straight-line method over 5 years. What will be the book value of the automobile at the end of the 3^{rd} year? (Assume the scrap value is \$0.)

Let V denote the value of the automobile at the end of each year.
Assume V is a linear function of t (i.e. the graph is a straight line)
We are given $V = 24{,}000$ when $t = 0$
and $V = 0$ when $t = 5$ (you hit scrap value in 5 years for this problem)

Picture:



So we have 2 points and we want to run a line through them.
We first calculate the slope:

$$m = \frac{V_1 - V_2}{t_1 - t_2} = \frac{24000 - 0}{0 - 5} = -\frac{24000}{5} = -4800$$

We now use the point slope form of the line with the point (0, 24000):

$V - V_1 = m(t - t_0)$ → V – 24000 = -4800 * (t – 0)
V – 24000 = -4800t
V = -4800t + 24000

Now we have the equation for the value of the automobile at any time t.
We are asked what the value will be at the 'end' of the 3^{rd} year. So put 3 in for t.

V = -4800t + 24000 → V = -4800*3 + 24000 = 9600

So the answer is $9600.

Section 1.3, Problem 38:

The quantity demanded for a certain brand of portable CD players is 200 units when the unit price is $90. The quantity demanded is 1200 units when the price is $40. Find the demand equation and sketch its graph.

Let p denote the unit price. Let x measure the quantity of units.
Recall p is a function of x (x is the variable).

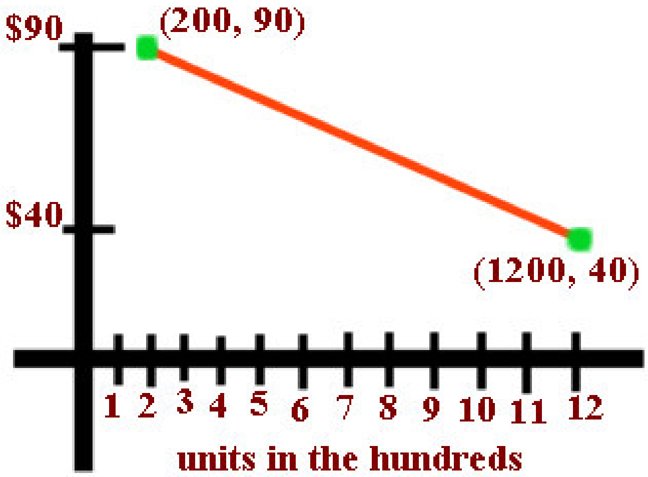
We have been given two points: (200, 90) and (1200, 40)
So like problem 16 we will first find the slope of our desired line:

$$m = \frac{p_1 - p_2}{x_1 - x_2} = \frac{90-40}{200-1200} = -\frac{50}{1000} = -\frac{1}{20}$$

And we use point-slope form to find our equation of the demand line:

$p - p_1 = m(x - x_1) \rightarrow$ p – 90 = -1/20 * (x – 200)
p – 90 = (-1/20)*x + 10
p = (-1/20)*x + 100

The graph would look like:



Section 1.4, Problem 22:

The demand equation for the Drake GPS Navigator is x + 4p – 800 = 0, where x is the quantity demanded per week and p is the wholesale unit price in dollars. The supply equation is x – 20p + 1000 = 0, where x is the quantity the supplier will make available in the market when the wholesale price is p dollars. Find the equilibrium quantity and the equilibrium price for the GPS Navigators.

You will likely be given a program for the calculators to solve this type of problem. What follows is how to work it out by hand:

Solve the system of equations:
Eq. 1: x + 4p – 800 = 0

Eq. 2: $x - 20p + 1000 = 0$

Solve equation 1 in terms of x:
$x + 4p - 800 = 0 \rightarrow \quad x = 800 - 4p$

Substitute $800 - 4p$ in for x in equation 2:
$x - 20p + 1000 = 0 \rightarrow (800 - 4p) - 20p + 1000 = 0$

Solve for p:

$$
\begin{aligned}
(800 - 4p) - 20p + 1000 = 0 \rightarrow \quad & 800 - 4p - 20p + 1000 = 0 \\
& -24p + 1800 = 0 \\
& -24p = -1800 \\
& p = -1800 / -24 \\
& p = 75
\end{aligned}
$$

Now put $p = 75$ into equation 1 and solve for x:

$$
\begin{aligned}
x + 4p - 800 = 0 \rightarrow \quad & x + 4*75 - 800 = 0 \\
& x + 300 - 800 = 0 \\
& x - 500 = 0 \\
& x = 500
\end{aligned}
$$

So the answer is x = 500 and p = 75

You may wish to put these values into equation 2 to make sure there are no errors (i.e. make sure the left side ends up equaling zero):
$x - 20p + 1000 = 500 - 20*75 + 1000 = 500 - 1500 + 1000 = 0$

Section 1.4, Problem 24:

The quantity demanded each month of Russo Espresso Makers is 250 when the unit price is \$140. The quantity demanded each month is 1000 when the unit price is \$110. The suppliers will market 750 espresso makers if the unit price is \$60 or lower. At a unit price of \$80, they are willing to make available 2250 units in the market. Both the demand and supply equations are known to be linear.

a. Find the demand equation

Notice the demand points given are: (250, \$140) and (1000, \$110)
As the demand function is linear all we need is the slope of the line through those 2 points and we can use the slope-point form of the equation.

The slope is:

$$m = \frac{p_1 - p_2}{x_1 - x_2} = \frac{140 - 110}{250 - 1000} = -\frac{30}{750} = -\frac{1}{25}$$

And we use point-slope form to find our equation of the demand line:

$p - p_1 = m(x - x_1) \rightarrow$ $p - 140 = -1/25 * (x - 250)$

$p - 140 = (-1/25)x + 10$

$p = (-1/25)x + 150$ ← **the demand equation**

b. Find the supply equation

Notice the supply points given are: (750, \$60) and (2250, \$80)
Again the supply equation is said to be linear so we do basically the same thing as above.

The slope is:

$$m = \frac{p_1 - p_2}{x_1 - x_2} = \frac{60 - 80}{750 - 2250} = \frac{20}{1500} = \frac{1}{75}$$

And we use point-slope form to find our equation of the supply line:

$p - p_1 = m(x - x_1) \rightarrow$ $p - 60 = 1/75 * (x - 750)$

$p - 60 = (1/75)x - 10$

$p = (1/75)x + 50$ ← **the supply equation**

c. Find the equilibrium quantity and price

Here we solve the system of equations created by the demand equation and the supply equation:

$p = (-1/25)x + 150$ ← the demand equation

$p = (1/75)x + 50$ ← the supply equation

------------------------- subtract the 2nd equation from the first

$0 = (-4/75)x + 100$

Solve for x:

$0 = (-4/75)x + 100 \rightarrow -100 = (-4/75)x$

$7500 = 4x$

$1875 = x$

Put 1875 in for x into the demand equation and solve for p:

$p = (-1/25)x + 150 \rightarrow p = (-1/25)*1875 + 150$

$p = -75 + 150$

$p = 75$

So the solution is x = 1875 units and p = \$75.

You may wish to check this by putting these values into the supply equation and check it comes out 'true:'

$p = (1/75)x + 50 \rightarrow \quad 75 = (1/75)*1875 + 50$
$75 = 25 + 50 = 75$ so it checks okay.

Section 1.5, Problem 14:

It is extremely likely you will be given a calculator program to work out this type of problem. It is MUCH easier with a calculator. I STRONGLY encourage you to learn how to do it using the calculator. I do NOT have the program.

The Social Security (FICA) wage base (in thousands of dollars) from 1996 to 2001 is given in the accompanying table:

Year:	1996	1997	1998	1999	2000	2001
Wage Base, y:	62.7	65.4	68.4	72.6	76.2	80.4

a. Find an equation of the least squares line for these data
(Let x = 1 represent the year 1996)

The needed table:

	x	y	x^2	xy
	1	62.7	1	62.7
	2	65.4	4	130.8
	3	68.4	9	205.2
	4	72.6	16	290.4
	5	76.2	25	381
	6	80.4	36	482.4
Sum	21	425.7	91	1552.5

Notice the number of data points = 6.

The normal equations are (compare color of numbers to see where they come from):
$6b + 21m = 425.7$
$21b + 91m = 1552.5$

And then you solve the above system to get m and b. Notice m and b will be your slope and y-intercept in the final equation (which of course will be in slope-intercept form).

So solve equation 1 for b:
$6b + 21m = 425.7 \rightarrow \quad 6b = 425.7 - 21m$
$b = 70.95 - 3.5m$

Put 70.95 – 3.5 into equation 2 for b and solve for m:
$21b + 91m = 1552.5 \rightarrow \quad 21*(70.95 - 3.5m) + 91m = 1552.5$
$1489.95 - 73.5m + 91m = 1552.5$
$1489.95 + 17.5m = 1552.5$

$17.5m = 62.55$
$m = 3.57429$

Put m = 3.57429 in for m into equation 1 and solve for b:

$6b + 21m = 425.7 \rightarrow 6b + 21*3.57429 = 425.7$

$6b + 75.06 = 425.7$

$6b = 425.7 - 75.06$

$6b = 350.64$

$b = 58.44$

So the least-squares line for the given set of data points is:

$y = mx + b \rightarrow$ **$y = 3.57429x + 58.44$**

b. Use the result of part (a) to estimate the FICA wage base in the year 2005

Recall 1996 → x = 1, so 2005 → x = 10

Think about it: 96 → 1
97 → 2
98 → 3
99 → 4
00 → 5
01 → 6
02 → 7
03 → 8
04 → 9
05 → 10

So put 10 in for x into our equation:

$y = 3.57429x + 58.44 \rightarrow y = 3.57429*10 + 58.44 \rightarrow$ **$y = 94.1829$**
