Finite Math Section 1_3
Solutions and Hints

by Brent M. Dingle

for the book:
Finite Mathematics, 7th Edition
by S. T. Tan.

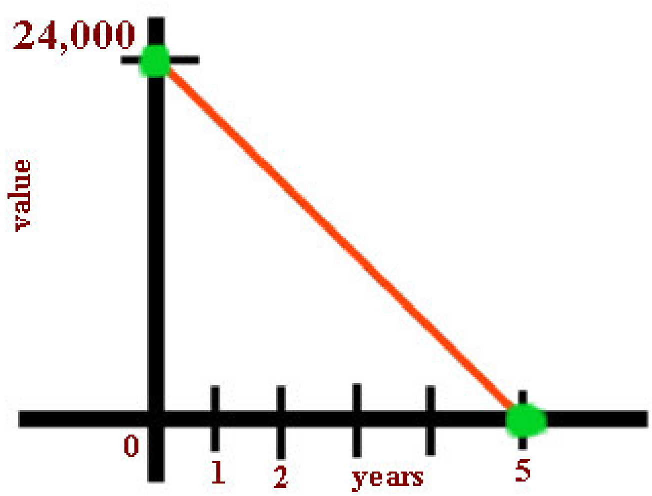
DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 1.3:

Problem 16:
An automobile purchased for use by the manager of a firm at a price of $24,000 is to be deprecated using the straight-line method over 5 years. What will be the book value of the automobile at the end of the 3rd year? (Assume the scrap value is $0.)

Let V denote the value of the automobile at the end of each year.
Assume V is a linear function of t (i.e. the graph is a straight line)
We are given V = 24,000 when t = 0
and V = 0 when t =5 (you hit scrap value in 5 years for this problem)

Picture:



So we have 2 points and we want to run a line through them.
We first calculate the slope:

$$m = \frac{V_1 - V_2}{t_1 - t_2} = \frac{24000 - 0}{0 - 5} = -\frac{24000}{5} = -4800$$

We now use the point slope form of the line with the point (0, 24000):

$V - V_1 = m(t - t_0)$ → V – 24000 = -4800 * (t – 0)
V – 24000 = -4800t
V = -4800t + 24000

Now we have the equation for the value of the automobile at any time t.
We are asked what the value will be at the 'end' of the 3^{rd} year. So put 3 in for t.

V = -4800t + 24000 → V = -4800*3 + 24000 = 9600

So the answer is $9600.

Problem 20:
AutoTime, a manufacturer of 24-hour variable timers, has a monthly fixed cost of $48,000 and a production cost of $8 for each timer manufactured. The timers sell for $14 each.

Assume that the number of timers made is the same as the number sold.

a. What is the cost function?
Cost = (fixed cost) + (cost per unit)*(number of units made)

C = 48000 + 8*x

b. What is the revenue function?
Revenue = (price per unit)*(number of units sold)

R = 14*x

c. What is the profit function?
Profit = Revenue – Cost

$P = R - C = (14*x) - (48000 + 8*x) =$ **$6*x - 48000$**

d. Compute the profit (loss) corresponding to production levels of 4000, 6000 and 10000 timers.

Just sub-in 4000 then 6000 then 10000 for x into the answer of part c:

$P(4000) = 6*4000 - 48000 =$ **-24000** (so a loss of $24000)

$P(6000) = 6*6000 - 48000 =$ **-12000** (so a loss of $12000)

$P(10000) = 6*10000 - 48000 =$ **12000** (so a profit of $12000)

Problem 22:

In 2000 National Textile installed a new machine in one of its factories at a cost of $250,000. The machine is depreciated linearly over 10 years with a scrap value of $10,000.

a. Find an expression for the machine's book value in the t^{th} year of use ($0 \leq t \leq 10$).

For these problems the value is always marked on the a-xis.
So you are actually given 2 points:
(0, $250000) and (10, $10000)

So the process is:
Find the slope, m, of the line through those 2 points.
Use the Point-Slope form $(y - y1) = m*(t - t1)$ to get the equation (where say $t1 = 0$ and $y1 = 250000$). Notice we used *t* instead of *x*.

$m = (10000 - 250000) / (10 - 0) = -24000$

So the equation is: $(y - 250000) = -24000*(t - 0)$
$y - 250000 = -24000*t$

$y = -24000*t + 250000$

b. Sketch the graph of the function of part (a).
You should be able to do this on your own.

c. Find the machine's book value in 2004.
Or rather find the machine's book value when $t = 4$.

Just plug $t = 4$ into the equation from part (a):
$y = -24000*t + 250000$
$y = -24000*4 + 250000$

$y = \$154,000$

d. Find the rate at which the machine is being depreciated.
This is just the slope of the line.
Notice it is important to state this correctly (as the negative implies depreciate).
The correct statement is:

The machine is depreciating at the rate of $24,000 per year.

Problem 30:
Entomologists have discovered that a linear relationship exists between the number of chirps of crickets of a certain species and the air temperature. When the temperature is 70° F, the crickets chirp at a rate of 120 chirps/minute. When the temperature is 80° F, they chirp at the rate of 160 chirps/min.

a. Find an equation giving the relationship between air temperature T and the number of chirps per minute N of the crickets.

In this problem you are given 2 points. We will let temperature be marked on the x-axis and the chirps per minute be on the y-axis. So the 2 points are:
(70° , 120 chirps) and (80° , 160 chirps) Notice this is in (T, N) form.

To get an equation for this first calculate the slope, m, of the line through the points and then use the point-slope form, $(N - N1) = m*(T - T1)$, to find the line.

slope $= m = (160 - 120) / (80 - 70) = 4$
So the equation is:
$N - 120 = 4*(T - 70)$
$N - 120 = 4*T - 280$

$N = 4*T - 160$

Notice you could also solve this for T and get $4T = N + 160$ or rather $T = (1/4)*N + 40$

b. Find N as a function of T and use this formula to determine the rate at which the crickets chirp when the temperature is 102° F.

Notice in part (a) we already have N as a function of T. So all we need to do is plug 102 in for T:

N = 4*T – 160
N = 4*102 – 160
# N = 248 chirps per minute.

Problem 38:
The quantity demanded for a certain brand of portable CD players is 200 units when the unit price is $90. The quantity demanded is 1200 units when the price is $40. Find the demand equation and sketch its graph.

Let p denote the unit price. Let x measure the quantity of units.
Recall p is a function of x (x is the variable).

We have been given two points: (200, 90) and (1200, 40)
So like problem 16 we will first find the slope of our desired line:

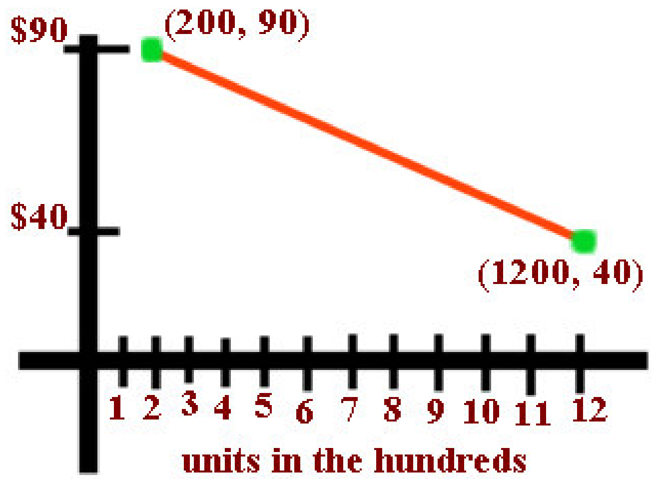
$$m = \frac{p_1 - p_2}{x_1 - x_2} = \frac{90 - 40}{200 - 1200} = -\frac{50}{1000} = -\frac{1}{20}$$

And we use point-slope form to find our equation of the demand line:

$p - p_1 = m(x - x_1)$ → $p - 90 = -1/20 * (x - 200)$
$p - 90 = (-1/20)*x + 10$

p = (-1/20)*x + 100

The graph would look like:



<u>Problem 46</u>:
producers will make 2000 refrigerators available when the unit price is $330. At a unit price of $390, 6000 refrigerators will be marketed.

<u>Find the equation relating the unit price of a refrigerator to the quantity supplied if the equation is known to be linear.</u>

Again price always goes on the y-axis.
So our two points are: (2000, $330) and (6000, $390).

The equation like most others in this section is found be first finding the slope, m, and then using point-slope form, $(p - p1) = m*(x - x1)$ to get the equation.

$m = (390 - 330) / (6000 - 2000) = 6 / 400 = 3 / 200$

The equation is thus:
$p - 330 = (3/200)*(x - 2000)$
$p - 330 = (3/200)*x - 30$

$p = (3/200)*x + 300$

<u>How many refrigerators will be marketed when the unit price is $450?</u>
For this use the above equation and plug $450 in for p. Then solve for x.

$450 = (3/200)*x + 300$
$150 = (3/200)*x$
$30000 = 3*x$

$10000 = x$

So when the price is $450 there producers will market 10,000 refrigerators.

<u>What is the lowest price at which **a** refrigerator will be marketed?</u>

In other words at what price will ONE refrigerator be made?

For that, simply plug 1 into the equation for x:
$p = (3/200)*x + 300$
$p = (3/200)*1 + 300$
$p = 0.015 + 300$

$p \approx \$300.02$