Finite Math Section 2_1
Solutions and Hints

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for the book:

Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 2.1:

Problem 8:
Determine if the system of linear equations has exactly one solution, infinitely many solutions or no solutions. Find all solutions when they exist:

Eq 1: $5x - 6y = 8$
Eq 2: $10x - 12y = 16$

Take equation 1 and solve for x → $5x = 8 + 6y$ → $x = 8/5 + (6/5)*y$
Substitute $8/5 + (6/5)*y$ in for x into Equation 2:
$10x - 12y = 16$ → $10*(8/5 + (6/5)*y) - 12y = 16$ and solve for y
$16 + 12y - 12y = 16$
$16 = 16$
$0 = 0$

From this (since $0 = 0$ always) we know that y can be any number.

When we have infinite solutions we will always want to express the solution in terms of t. So we must express x in terms of t and we must express y in terms of t.

So let's assume we 'know' that the value of $y = t$ (t is a fixed number).
We then substitute t in for y into equation 1:
$5x - 6y = 8$ → $5x - 6t = 8$ and solve for x in terms of t
→ $5x = 8 + 6t$
→ $x = (8/5) + (6/5)*t$

Now we have x in terms of t and y in terms of t. Specifically we have:
$x = (8/5) + (6/5)*t$
$y = t$

So our answer is:

There are infinitely many solutions in the form of (8/5 + (6/5)*t, t)

Some professors will refer to the above as being in parametric form.

Problem 14:
Determine the value of k for which the system of linear equations:

Eq 1: $3x + 4y = 12$
Eq 2: $x + ky = 4$

has infinitely many solutions. Then find all solutions corresponding to this value of k.

So solve for x using Eq 1:
$3x + 4y = 12 \rightarrow 3x = 12 - 4y \rightarrow x = 4 - (4/3)y$

Substitute $4 - (4/3)y$ in for x into Eq. 2:
$x + ky = 4 \rightarrow (4 - (4/3)y) + ky = 4$ solve in terms of y
$\rightarrow 4 - (4/3)y + ky = 4$
$\rightarrow -(4/3)y + ky = 0$ (so here everything is in terms of y)
$\rightarrow ky = (4/3)y$ (now we need a value of k to make the equality always true)
$\rightarrow k = 4/3$ (we divided both sides by y)

From this we know if we let $k = 4/3$ then y could have any value because when we solve the system of equations we would always get $4/3 = 4/3$ or $0 = 0$.
So if $k = 4/3$ then there are an infinite number of solutions to the system of equations.

Because there are an infinite number of solutions we need to express all the solutions in terms of t. So we need to express x in terms of t and y in terms of t.

We will let y be any number. We show this by saying $y = t$.
We then need to get x in terms of t, so we substitute t in for y into Eq 1:
$3x + 4y = 12 \rightarrow 3x + 4t = 12$ (now solve for x in terms of t)
$\rightarrow 3x = 12 - 4t$
$\rightarrow x = 4 - (4/3)t$

And we have x and y in terms of t:
$x = 4 - (4/3)t$
$y = t$

Thus when k = 4/3 we have infinite solutions to the system of equations all in the form of: (4 – (4/3)t,   t)

Problem 18:
Kelly Fisher has a total of $30,000 invested in two municipal bonds that have yields of 8% and 10% interest per year, respectively. If the interest Kelly receives from the bonds in a year is $2640, how much does she have invested in each bond?

NOTICE: This is a SETUP but do NOT solve (you must solve it in section 2.2)

Let x = amount invested in bond 1 (with 8% interest).
Let y = amount invested in bond 2 (with 10% interest).

The total invested is $30,000 so we have
Eq 1: $x + y = 30000$

The interest in one year from bond 1 is: $0.08*x$
The interest in one year from bond 2 is: $0.10*y$
The total interest gained in a year is $2640 so we have
Eq 2: $0.08x + 0.10y = 2640$

This gives us the system of equations as follows:

Eq 1: $x + y = 30000$
Eq 2: $0.08x + 0.10y = 2640$

Problem 24:
A theater has a seating capacity of 900 and charges $2 for children, $3 for students and $4 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. The receipts totaled $2800. How many children attended the show?

NOTICE: This is a SETUP but do NOT solve (you must solve it in section 2.2)

Let x = number of children attending
Let y = number of students attending

Let z = number of adults attending

The theater was full and has a capacity of 900 so we have

Eq 1: $x + y + z = 900$

The money from children attending $= 2x$
The money from students attending $= 3y$
The money from adults attending $= 4z$
The total money was \$2800 so we have

Eq 2: $2x + 3y + 4z = 2800$

Lastly we know the number of adults was half that of the children and students combined, so we have:

Eq 3: $z = \frac{1}{2}(x + y)$

And that gives us a system of 3 equations and 3 unknowns:

Eq 1: $\mathbf{x + y + z = 900}$
Eq 2: $\mathbf{2x + 3y + 4z = 2800}$
Eq 3: $\mathbf{z = \frac{1}{2}(x + y) = 0.5x + 0.5y}$