Finite Math Section 2_2
Solutions and Hints

by Brent M. Dingle

for the book:

Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 2.2:

This entire section is most often skimmed or skipped because of the prevalent use of calculators. However it is useful to learn how to solve these problems with the calculator and how to set up the system of equations for the story problems. Occasionally the instructors may talk about pivoting. The definitions are also important to learn – such as augmented matrix and row-reduced form. You should ask your instructor if you will ever be required to perform a Gauss-Jordan Elimination method "by hand."

Problem 24:
Pivot around the element in row 2 column 1:

Pivot in this context means:
1. make the circled element be 1 and
2. zero out all other elements in its column.

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 2 & 4 & 8 & 6 \\ -1 & 2 & 3 & 4 \end{array}\right] \xrightarrow{1/2*R2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 1 & 2 & 4 & 3 \\ 0 & 4 & -1 & 3 \end{array}\right] \xrightarrow{R1-R2}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -2 & 1 \\ 1 & 2 & 4 & 3 \\ -1 & 2 & 3 & 4 \end{array}\right] \xrightarrow{R3+R2} \left[\begin{array}{ccc|c} 0 & 1 & -2 & 1 \\ 1 & 2 & 4 & 3 \\ 0 & 4 & 7 & 7 \end{array}\right]$$ **← there is the answer**

Problems 35 to 50:

Unless your instructor specifically says that you are going to have to perform a Gauss-Jordan reduction BY HAND on your tests – these can be skipped because there is a much simpler way to solve them with your calculator.

The Gauss-Jordan method is introduced to show you how matrices and row operations came to be a method for solving systems of linear equations. It is worth learning, but if all you want to do is solve the systems there are easier ways.

Problem 54:

Kelly Fisher has a total of $30,000 invested in two municipal bonds that have yields of 8% and 10% interest per year, respectively. If the interest Kelly receives from the bonds in a year is $2640, how much does she have invested in each bond?

Notice this relates to problem 18 in 2.1.

Let x = amount invested in bond 1 (with 8% interest).
Let y = amount invested in bond 2 (with 10% interest).

The total invested is $30,000 so we have

Eq 1: $x + y = 30000$

The interest in one year from bond 1 is: $0.08*x$
The interest in one year from bond 2 is: $0.10*y$
The total interest gained in a year is $2640 so we have

Eq 2: $0.08x + 0.10y = 2640$

We now have 2 equations and 2 unknowns so we can solve the system for x and y.

First solve Eq 1 for x:
$x + y = 30000 \rightarrow x = 30000 - y$

Now sub in 30000 – y for x into Eq 2:
$0.08x + 0.10y = 2640 \rightarrow 0.08*(30000 - y) + 0.10y = 2640$ (and solve for y)
$\rightarrow 2400 - 0.08y + 0.10y = 2640$
$\rightarrow 0.02y = 240$
$\rightarrow y = 12000$

Now sub 12000 in for y into Equation 1 and solve for x:
$x + y = 30000 \rightarrow x + 12000 = 30000$
$\rightarrow x = 18000$

So she has $18000 invested in bond 1 and $12000 invested in bond 2.

Problem 60:
A theater has a seating capacity of 900 and charges $2 for children, $3 for students and $4 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. The receipts totaled $2800. How many children attended the show?

Notice this relates to problem 24 in section 2.1.

Let x = number of children attending
Let y = number of students attending
Let z = number of adults attending

The theater was full and has a capacity of 900 so we have
Eq 1: $x + y + z = 900$

The money from children attending = 2x
The money from students attending = 3y
The money from adults attending = 4z
The total money was $2800 so we have
Eq 2: $2x + 3y + 4z = 2800$

Lastly we know the number of adults was half that of the children and students combined, so we have:
Eq 3: $z = \frac{1}{2}(x + y)$

And that gives us a system of 3 equations and 3 unknowns:
Eq 1: $x + y + z = 900$
Eq 2: $2x + 3y + 4z = 2800$
Eq 3: $z = \frac{1}{2}(x + y) = 0.5x + 0.5y$

Notice you should use the Gauss Jordan method to solve this – it will be easier. And if possible using the TI-83's rref will prove to be even faster. However below is the 'hard' way to do it (and the way most students will solve it if asked).

Since in Eq 3, we already have z in terms of x and y let's put that into Eq 1:

$x + y + z = 900 \rightarrow x + y + (0.5x + 0.5y) = 900$ (and solve for y)

$\rightarrow 1.5x + 1.5y = 900$

$\rightarrow 1.5y = 900 - 1.5x$

$\rightarrow y = 600 - x$ * remember this

Call that Eq *: $y = 600 - x$

Now sub in 0.5x and 0.5y in for x into Eq 2:

$2x + 3y + 4z = 2800 \rightarrow 2x + 3y + 4*(0.5x + 0.5y) = 2800$

$\rightarrow 2x + 3y + 2x + 2y = 2800$

$\rightarrow 4x + 5y = 2800$ * now sub in $600 - x$ for y

$\rightarrow 4x + 5*(600 - x) = 2800$

$\rightarrow 4x + 3000 - 5x = 2800$

$\rightarrow -x + 3000 = 2800$

$\rightarrow -x = -200$

$\rightarrow x = 200$

Put 200 in for x into Eq *:

$y = 600 - x \rightarrow y = 600 - 200 \rightarrow y = 400$

And putting 400 in for y, and putting 200 in for x into Eq 3 we get:

$z = 0.5x + 0.5y \rightarrow z = 0.5*200 + 0.5*400$

$\rightarrow z = 100 + 200$

$\rightarrow z = 300$

So we have x = 200, y = 400 and z = 300

Thus our answer is:

200 children attended the show

Later you will learn how to solve these systems using the TI-83's rref function (under matrix functions)

Problem 66:

A manufacturer of women's blouses makes three types of blouses: sleeveless, short sleeve and long sleeve. The time (in minutes) required by each department to produce a dozen blouses of each type is shown in the below table:

	Sleeveless	Short Sleeve	Long Sleeve
Cutting	9	12	15
Sewing	22	24	28
Packaging	6	8	8

The cutting, sewing and packaging departments have available a maximum of 80, 160 and 48 labor HOURS respectively per day. How many dozens of each type of blouse can be produced each day if the plant is operated at full capacity.

Let x = number of dozens of sleeveless shirts produced
Let y = number of dozens of short sleeve shirts produced
Let z = number of dozens of long sleeve shirts produced

Maximum time for cutting available is 80 hours = 4800 minutes
Minutes spent cutting sleeveless shirts = 9x
Minutes spent cutting short sleeve shirts = 12y
Minutes spent cutting long sleeve shirts = 15z
So we have:
Eq 1: $9x + 12y + 15z = 4800$

Maximum time for sewing available is 160 hours = 9600 minutes
Minutes spent sewing sleeveless shirts = 22x
Minutes spent sewing short sleeve shirts = 24y
Minutes spent sewing long sleeve shirts = 28z
So we have:
Eq 2: $22x + 24y + 28z = 9600$

Maximum time for packaging available is 48 hours = 2880 minutes
Minutes spent packaging sleeveless shirts = 6x
Minutes spent packaging short sleeve shirts = 8y
Minutes spent packaging long sleeve shirts = 8z
So we have:
Eq 3: $6x + 8y + 8z = 2880$

So the system of equations we must solve is:
Eq 1: $9x + 12y + 15z = 4800$
Eq 2: $22x + 24y + 28z = 9600$
Eq 3: $6x + 8y + 8z = 2880$

We now solve using the Gauss-Jordan Elimination method (as detailed on page 88 of your book):

$$\left[\begin{array}{ccc|c} 9 & 12 & 15 & 4800 \\ 22 & 24 & 28 & 9600 \\ 6 & 8 & 8 & 2880 \end{array}\right] \xrightarrow{1/9*R1} \left[\begin{array}{ccc|c} 1 & 4/3 & 5/3 & 1600/3 \\ 22 & 24 & 28 & 9600 \\ 6 & 8 & 8 & 2880 \end{array}\right] \xrightarrow[R3-6*R1]{R2-22*R1}$$

$$\left[\begin{array}{ccc|c} 1 & 4/3 & 5/3 & 1600/3 \\ 0 & -16/3 & -26/3 & -6400/3 \\ 0 & 0 & -2 & -320 \end{array}\right] \xrightarrow{-3/16*R2} \left[\begin{array}{ccc|c} 1 & 4/3 & 5/3 & 1600/3 \\ 0 & 1 & 13/8 & 400 \\ 0 & 0 & -2 & -320 \end{array}\right] \xrightarrow{R1-4/3*R2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 13/8 & 400 \\ 0 & 0 & -2 & -320 \end{array}\right] \xrightarrow{-1/2*R3} \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 13/8 & 400 \\ 0 & 0 & 1 & 160 \end{array}\right] \xrightarrow[R2-13/8*R3]{R1+1/2*R3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 80 \\ 0 & 1 & 0 & 140 \\ 0 & 0 & 1 & 160 \end{array}\right]$$

So we see that x = 80, y = 140, z = 160

Do a quick check using Eq 1:
$9x + 12y + 15z = 4800 \rightarrow 9*80 + 12*140 + 15 * 160 = 4800$

So we conclude:

80 dozen sleeveless shirts are produced,
140 dozen short sleeve shirts are produced and
160 dozen long sleeve shirts are produced.
