Finite Math Section 2_6 Solutions and Hints

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for the book:

Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!! This is designed to assist you in the event you get stuck. If you do not do the work you will NOT pass the tests.

Section 2.6:

This section is all about finding the inverses of matrices. For the most part in this class you will always find the inverse using the calculator (notice you must use the x^{-1} button on the calculator → [A]^-1 will NOT work on the TI-83)

The most significant thing in this section is you can now solve systems by putting them into A**x** = **b** form by calculating A inverse.

Recall that if you wanted to solve 3x = 5 you would multiply both sides by 3^{-1} (i.e. 1/3).

The same can be done to solve A**x** = **b**, multiply both sides by A^{-1} and you get $\mathbf{x} = A^{-1}\mathbf{b}$.

Problem 18:

Using an equivalent matrix equation solve the system of equations:

Eq 1: 2x + 3y = 5
Eq 2: 3x + 5y = 8

Putting it into Ax = b form:

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Notice either from problem 6 or by a calculator (or both) you know that the inverse of the matrix A is:

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$$

And we know by algebra, $x = A^{-1}b$:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}`$$

Thus we have the solution of the system of equations is **x = 1 and y = 1**

Problem 26:

Write the system of equations as a matrix equation and solve the system by using the inverse of the coefficient matrix:

$3x - 2y = b_1$
$4x + 3y = b_2$

a. where $b_1 = -6$ and $b_2 = 10$
b. where $b_1 = 3$ and $b_2 = -2$

To solve both part (a) and (b) in the least amount of time it is easiest to set up the problem with the b_1 and b_2 as part of the matrix system:

First we get everything into Ax = b form:

$$\begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Then we migrate everything into $x = A^{-1}b$ form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 3/17 & 2/17 \\ -4/17 & 3/17 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} (3/17)*b_1 + (2/17)*b_2 \\ (-4/17)*b_1 + (3/17)*b_2 \end{bmatrix}$$

Thus we have:

$x = (3/17)*b_1 + (2/17)*b_2$
$y = (-4/17)*b_1 + (3/17)*b_2$

So for part (a) where $b_1 = -6$ and $b_2 = 10$ we get:

$x = (3/17)*(-6) + (2/17)*10$
$y = (-4/17)*(-6) + (3/17)*10$

Or rather:

x = 2/17
y = 54/17

And for part (b) where $b_1 = 3$ and $b_2 = -2$ we get:

x = (3/17)*(3) + (2/17)*(-2)
y = (-4/17)*(3) + (3/17)*(-2)

,
Or rather:

x = 5/17
y = -18/17

Problem 36:

Rainbow Harbor Cruises charges $8/adult and $4/child for a round-trip ticket. The records show that on a certain weekend, 1000 people took the cruise on Saturday and 800 people took the cruise on Sunday. The total receipts for Saturday were $6400 and the ‘ total receipts for Sunday were $4800. Determine how many adults and children took the cruise on Saturday and Sunday.

This is actually 2 problems – one concerns Saturday, the other Sunday

Let c = number of children on the cruise on Saturday
Let a = number of adults on the cruise on Saturday

1000 people took the cruise on Saturday, so
Eq 1: c + a = 1000

The total money made on Saturday was $6400
The income from children on Sat = 4c
The income from adults on Sat = 8a
So we have:
Eq 2: 4c + 8a = 6400

,
Our system of equations is thus:
Eq 1: c + a = 1000
Eq 2: 4c + 8a = 6400

Putting this into Ax = b form we get:

$$\begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} c \\ a \end{bmatrix} = \begin{bmatrix} 1000 \\ 6400 \end{bmatrix}$$

We now put it into x = A^{-1}b form:

$$\begin{bmatrix} c \\ a \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 1000 \\ 6400 \end{bmatrix} = \begin{bmatrix} 2 & -1/4 \\ -1 & 1/4 \end{bmatrix} \begin{bmatrix} 1000 \\ 6400 \end{bmatrix} = \begin{bmatrix} 400 \\ 600 \end{bmatrix}$$

So we know there were 400 children and 600 adults on Saturday.

You can work the part for Sunday in an similar fashion.
