Finite Math Section 3_2
Solutions and Hints

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for the book:
Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 3.2:

For this section all you do is formulate the system, you do not solve it until the next section. These are extremely popular types of problems for this class (and will show up in other business and math and engineering classes). Learn how to set up and solve these. Do as many as possible.

Notice only some of these appear in the next section.
Specifically:

Problem number in Section 3.2	Problem number in Section 3.3
1	30
2	33
3	34
4	35
5	36
9	39
10	40
12	41
13	43
14	44
15	45

Problem 1:
A company manufacturers two products, A and B, on two machines I and II. It has been determined that the company will realize a profit of $3 on each unit of

product A and a profit of $4 on each unit of product B. To manufacture a unit of product A requires 6 minutes on machine I and 5 minutes on machine II. To manufacture a unit of product B requires 9 minutes on machine I and 4 minutes on machine II. There are 5 hours of machine time available on machine I and 3 hours of machine time available on machine II in each work shift. How many units of each product should be produced in each shift to maximize the company's profit?

First let's make a table to organize our information (this really does help):
Notice we convert all the times to minutes.

	Product A	**Product B**	**Time Available**
Machine I	6 minutes	9 minutes	300 minutes
Machine II	5 minutes	4 minutes	180 minutes
Profit / Unit	$3	$4	

Let x = number of units of product A made
Let y = number of units of product B made

So our objective is to maximize the profit function.
Profit = income from Product A + income from Product B so

$$\mathbf{P = 3x + 4y}$$

The total time available on machine I is 300 minutes
(which we may or may not completely use).
The time used creating product A on machine I = 6x
The time used creating product B on machine I = 5y
So

$$\mathbf{6x + 9y \le 300}$$

The total time available on machine II is 180 minutes
(which we may or may not completely use).
The time used creating product A on machine II = 9x
The time used creating product B on machine II = 4y
So

$$\mathbf{5x + 4y \le 180}$$

Notice also we cannot produce negative amounts of the products so we also have:

$$\mathbf{x \ge 0}$$
$$\mathbf{y \ge 0}$$

In summary:

We are to maximize our objective function:

$$P = 3x + 4y$$

Subject to the constraints:

$$6x + 9y \le 300$$
$$5x + 4y \le 180$$
$$x \ge 0$$
$$y \ge 0$$

Problem 4:

Madison Finance has a total of $20 million earmarked for homeowner and auto loans. On average, homeowner loans have a 10% annual rate of return, whereas auto loans yield a 12% annual rate of return. Management has also stipulated that the total amount of homeowner loans should be greater than or equal to four times the total amount of automobile loans. Determine the total amount of loans of each type Madison should extend to each category in order to maximize its returns.

Let x = amount of homeowner loans
Let y = amount of automobile loans

We are to maximize the returns (the profit).
The profit from home loans = 0.10x
The profit from auto loans = 0.12y
So we are to maximize the objective function:

$$P = 0.10x + 0.12y$$

We are told to the total amount earmarked is $20,000,000
(so we may spend at most that much)
So we have:

$$x + y \le 20{,}000{,}000$$

We also know home loans are to be ≥ 4*(auto loans).
So we have:

$$x \ge 4y$$

And we recognize we cannot have negative home loan or auto loan totals so:

$$x \ge 0$$
$$y \ge 0$$

Thus our setup is:

We are to maximize the objective function:

$$P = 0.10x + 0.12y$$

Under the constraints of:

$$x + y \leq 20000000$$
$$x \geq 4y$$
$$x \geq 0$$
$$y \geq 0$$

Problem 10:

Deluxe River Cruises operates a fleet of river vessels. The fleet has two types of vessels: A type-A vessel has 60 deluxe cabins and 160 standard cabins, whereas a type-B vessel has 80 deluxe cabins and 120 standard cabins. Under a charter agreement with Odyssey Travel Agency, Deluxe River Cruises is to provide Odyssey with a minimum of 360 deluxe and 680 standard cabins for their 15-day cruise in May. It costs $44,000 to operate a type-A vessel and $54,000 to operate a type-B vessel for that period. How many of each type of vessel should be used in order to keep the operating costs to a minimum.

A table will help for this one:

	Type A	Type B	Minimum Needed
Deluxe Cabins	60	80	360
Standard Cabins	160	120	680
Cost	44000	54000	

Let x = number of type A ships
Let y = number of type B ships

We want to minimize cost. So our objective function is:

$$C = 44000x + 54000y$$

There must be at least 360 deluxe cabins available so:

$$60x + 80y \geq 360$$

There must be at least 680 standard cabins available so:

$$160x + 120y \geq 680$$

And of course we cannot have negative numbers of ships so:

$\mathbf{x \geq 0}$

$\mathbf{y \geq 0}$

And the system is:

Minimize the objective function:

$$\mathbf{C = 44000x + 54000y}$$

Under the constraints:

$$\mathbf{60x + 80y \geq 360}$$

$$\mathbf{160x + 120y \geq 680}$$

$$\mathbf{x \geq 0}$$

$$\mathbf{y \geq 0}$$

Problem 15:

TMA manufacturers 19 inch color television picture tubes in two separate locations, location I and location II. The output at location I is at most 6000 tubes/month, whereas the output at location II is at most 5000 tubes/month. TMA is the main supplier of picture tubes to Pulsar Corporation, its holding company, which has priority in having all its requirements met. In a certain month, Pulsar placed orders for 3000 and 4000 picture tubes to be shipped to two of its factories located in city A and city B, respectively. The shipping costs (in dollars) per picture tube from the two TMA plants to the Pulsar factories are as follows:

	To Pulsar Factories in City A	To Pulsar Factories in City B
From TMA Location I	$3	$2
From TMA Location II	$4	$5

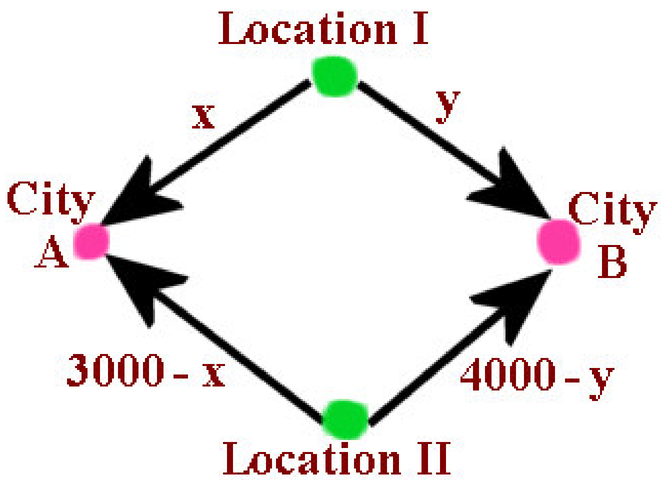
Find a shipping schedule that meets the requirements of both companies while keeping costs to a minimum.

For this a picture showing how stuff moves will help.

Let x = the number of tubes sent from Location I to City A.
Note that city A needs 3000 tubes.
So the number of tubes sent from Location II to City A must be 3000- x.
(i.e. the number shipped from Location I and Location II to city A must add up to 3000)

Let y = the number of tubes sent from Location I to city B.
Note that city needs 4000 tubes.
So the number of tubes sent from Location II to City B must be 4000 – y.

The picture looks like this:



Our objective function to MINIMIZE is cost.
The cost of tubes leaving Location I = 3x + 2y
The cost of tubes leaving Location II = 4*(3000 – x) + 5*(4000 – y)
= 32000 – 4x – 5y
So our total cost is:

$C = (3x + 2y) + (32000 - 4x - 5y)$

$\mathbf{C = 32000 - x - 3y}$

Since Location I can only produce 6000 tubes per month we have the constraint:

$\mathbf{x + y \leq 6000}$

Since Location II can only produce 5000 tubes per month we also have:

$(3000 - x) + (4000 - y) \leq 5000$, or simplifying

$7000 - x - y \leq 5000$

$2000 \leq x + y$

$\mathbf{x + y \geq 2000}$

We also know that we cannot ship negative amounts of tubes so we know:

$\mathbf{x \geq 0}$

$\mathbf{y \geq 0}$

$3000 - x \geq 0 \rightarrow 3000 \geq x \rightarrow \mathbf{x \leq 3000}$

$4000 - y \geq 0 \rightarrow 4000 \geq y \rightarrow \mathbf{y \leq 4000}$

And so the system we are to solve is:

Minimize the objective function:

$C = 32000 - x - 3y$

Subject to the constraints:

$x + y \leq 6000$

$x + y \geq 2000$

$x \geq 0$

$y \geq 0$

$x \leq 3000$

$y \leq 4000$