Finite Math Section 3_3
Solutions and Hints

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for the book:
Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 3.3:
These are extremely popular types of problems for this class (and will show up in other business and math and engineering classes). Learn how to set up and solve these. Do as many as possible.

Be certain to understand the Method of Corners, page 191.

Notice some of these were 'formulated' in the previous section.
Specifically:

Problem number in Section 3.2	Problem number in Section 3.3
1	30
2	33
3	34
4	35
5	36
9	39
10	40
12	41
13	43
14	44
15	45

Problem 30:
A company manufacturers two products, A and B, on two machines I and II. It has been determined that the company will realize a profit of $3 on each unit of

product A and a profit of $4 on each unit of product B. To manufacture a unit of product A requires 6 minutes on machine I and 5 minutes on machine II. To manufacture a unit of product B requires 9 minutes on machine I and 4 minutes on machine II. There are 5 hours of machine time available on machine I and 3 hours of machine time available on machine II in each work shift. How many units of each product should be produced in each shift to maximize the company's profit?

First let's make a table to organize our information (this really does help):
Notice we convert all the times to minutes.

	Product A	**Product B**	**Time Available**
Machine I	6 minutes	9 minutes	300 minutes
Machine II	5 minutes	4 minutes	180 minutes
Profit / Unit	$3	$4	

Let x = number of units of product A made
Let y = number of units of product B made

So our objective function is to maximize profit.
Profit = income from Product A + income from Product B so

$P = 3x + 4y$

The total time available on machine I is 300 minutes
(which we may or may not completely use).
The time used creating product A on machine I = 6x
The time used creating product B on machine I = 9y
So

$6x + 9y \leq 300$

The total time available on machine II is 180 minutes
(which we may or may not completely use).
The time used creating product A on machine II = 5x
The time used creating product B on machine II = 4y
So

$5x + 4y \leq 180$

Notice also we cannot produce negative amounts of the products so we also have:

$x \geq 0$
$y \geq 0$

In summary:

We are to maximize our objective function:

$P = 3x + 4y$

Subject to the constraints:

$6x + 9y \leq 300$
$5x + 4y \leq 300$
$x \geq 0$
$y \geq 0$

So graph all the constraint lines (this should be 'old hat' by now).

<u>For the line 6x + 9y = 300,</u>

Let x = 0, solve for y, get y = 100/3 → first point on the line is (0, 100/3)
Let y = 0, solve for x, get x = 50 → second point on the line is (50, 0)
Draw the line.
Pick the point (0, 0) and put it into $6x + 9y \leq 300$ → $0 + 0 \leq 300$ is true, so the 'true side' is below the line.

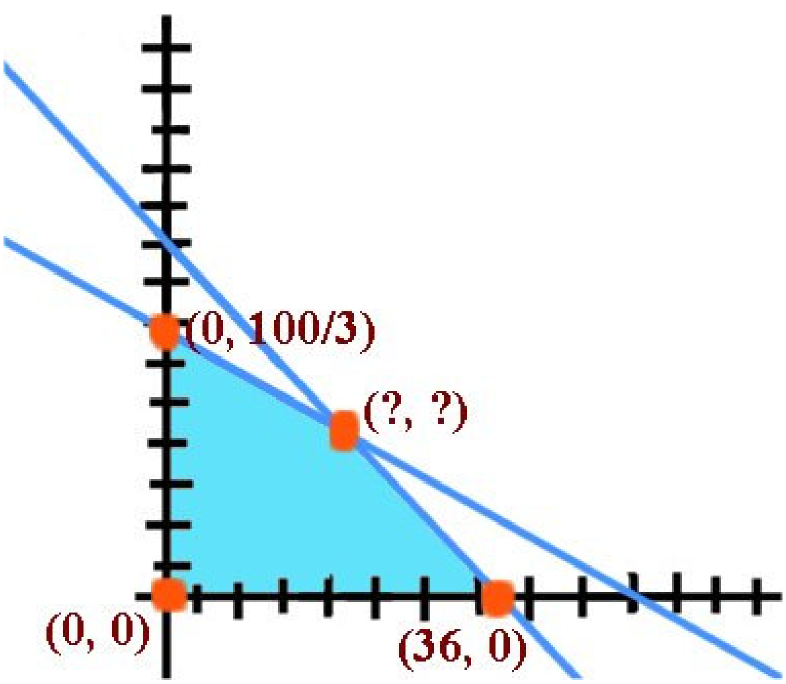
<u>For the line 5x + 4y = 180,</u>

Let x = 0, solve for y, get y = 45 → first point on the line is (0, 45)
Let y = 0, solve for x, get x = 36 → second point on the line is (36, 0)
Draw the line.
Pick the point (0, 0) and put it into $5x + 4y \leq 180$ → $0 + 0 \leq 180$ is true, so the 'true side' is below the line.

For $x \geq 0$ the true side is everything to the right of the y-axis.
For $y \geq 0$ the true side is everything above the x-axis.

So your picture should look something like (hash marks are in increments of 5):

NOTE: Most instructors at A&M will have you shade the FALSE side, in the below the true sides were shaded (as the book does it).



You now need to determine the intersection point of the lines:

Eq 1: $6x + 9y = 300$

Eq 2: $5x + 4y = 180$

Solving for y in Eq 2 → $4y = 180 – 5x$ → $y = 45 – (5/4)x$

Sub that in for y into Eq 1 and solve for x:

$6x + 9y = 300$ → $6x + 9*(45 – (5/4)x) = 300$

→ $6x + 405 – (45/4)x = 300$

→ $(-21/4)x = -105$

→ $x = 20$

Put that value in for x into Eq 2 and solve for y:

$5x + 4y = 180$ → $5*20 + 4y = 180$ → $4y = 80$ → $y = 20$

So the unknown ‘corner’ is actually (20, 20).

Now all we need to do is put all our corner points into our objective function: $P = 3x + 4y$
We will use a table for this:

Point	**Value of P = 3x + 4y**
(0, 0)	0
(36, 0)	108
(0, 100/3)	400/3 = 133 and 1/3
(20, 20)	140

And we pick point (20, 20) as the one that gives us the greatest profit of $140.

So our answer is:

We should produce 20 units of product A and 20 units of product B to get a maximum profit of $140.

Problem 35:

Madison Finance has a total of $20 million earmarked for homeowner and auto loans. On average, homeowner loans have a 10% annual rate of return, whereas auto loans yield a 12% annual rate of return. Management has also stipulated that the total amount of homeowner loans should be greater than or equal to four times the total amount of automobile loans. Determine the total amount of loans of each type Madison should extend to each category in order to maximize its returns.

Let x = amount of homeowner loans
Let y = amount of automobile loans

We are to maximize the returns (the profit).
The profit from home loans = 0.10x
The profit from auto loans = 0.12y
So we are to maximize the objective function:

P = 0.10x + 0.12y

We are told to the total amount earmarked is $20,000,000
(so we may spend at most that much)
So we have:

x + y ≤ 20,000,000

We also know home loans are to be ≥ 4*(auto loans).
So we have:

x ≥ 4y

And we recognize we cannot have negative home loan or auto loan totals so:

x ≥ 0
y ≥ 0

Thus our setup is:

We are to maximize the objective function:

P = 0.10x + 0.12y

Under the constraints of:

x + y ≤ 20000000
x ≥ 4y
x ≥ 0
y ≥ 0

So we need to graph the constraint lines.
NOTE: Most instructors at A&M will have you shade the FALSE side.

For the line x + y = 20,000,000:

Let x = 0, and solve for y,
So we get y = 20,000,000 → first point on the line is (0, 20000000)
Let y = 0, solve for x,
And we get x = 20,000,000 → second point on the line is (20000000, 0)
Draw the line through (0, 20000000) and (20000000, 0)
Pick the point (0, 0) and put it into x + y ≤ 20,000,000
→ 0 + 0 ≤ 20,000,000 is true, so the 'true side' is 'below' the line.

<u>For the line x = 4y:</u>

Let x = 0, and solve for y,
So we get 0 = 4y → 0 = y
So our first point on this line is (0, 0)
Let y = 1000000, solve for x,
And we get x = 4*1000000 → x = 4000000
So our second point on the line is (4000000, 1000000)
Draw the line through the points (0, 0) and (4000000, 1000000)
Notice we cannot pick (0, 0) as a test point as (0, 0) is on the line.
So pick the point (0, 5000000) (which is 'above' the line) and put it into x ≥ 4y
→ 0 ≥ 20000000 is FALSE, so the true side is 'below' the line.

<u>For x ≥ 0 the true side is everything to the right of the y-axis.</u>

<u>For y ≥ 0 the true side is everything above the x-axis.</u>

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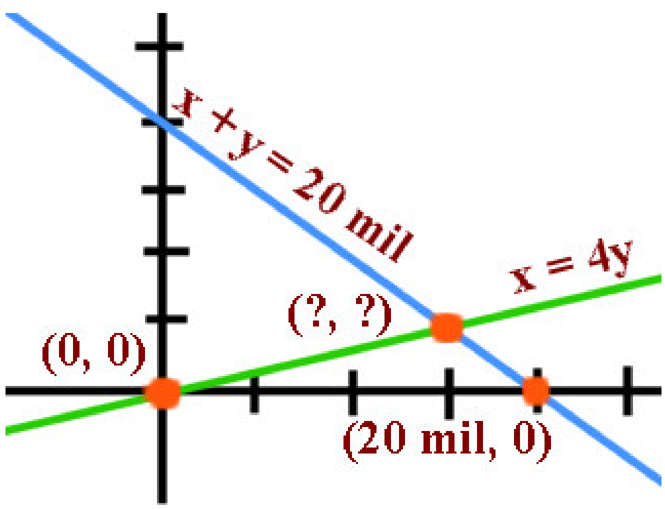
Hash marks are in increments of 5 million.



You now need to determine the intersection point of the lines:
Eq 1: x + y = 20000000
Eq 2: x = 4y

Substitute 4y in for x into Eq 1 and solve for y:
x + y = 20000000 → 4y + y = 20000000
→ 5y = 20000000
→ y = 4000000

Put that value in for y into Eq 2 and solve for x:
x = 4y → x = 4*4000000 → x = 16000000

So the unknown 'corner' is actually (16000000, 4000000).

Now all we need to do is put all our corner points into our objective function:
P = 0.10x + 0.12y

We will use a table for this:

Point	Value of P = 0.10x + 0.12y
(0, 0)	0
(20000000, 0)	2,000,000
(16 000 000, 4 000 000)	2,080,000

And we pick point (16 000 000, 4 000 000) which gives a profit of $2.08 million.

So our answer is:

Extend $16 million in homeowner loans and $4 million in automobile loans to yield a maximal profit of $2.8 million.

Problem 10:
Deluxe River Cruises operates a fleet of river vessels. The fleet has two types of vessels: A type-A vessel has 60 deluxe cabins and 160 standard cabins, whereas a type-B vessel has 80 deluxe cabins and 120 standard cabins. Under a charter agreement with Odyssey Travel Agency, Deluxe River Cruises is to provide Odyssey with a minimum of 360 deluxe and 680 standard cabins for their 15-day cruise in May. It costs $44,000 to operate a type-A vessel and $54,000 to operate a type-B vessel for that period. How many of each type of vessel should be used in order to keep the operating costs to a minimum.

A table will help for this one:

	Type A	Type B	Minimum Needed
Deluxe Cabins	60	80	360
Standard Cabins	160	120	680
Cost	44000	54000	

Let x = number of type A ships
Let y = number of type B ships

We want to minimize cost. So our objective function is:

C = 44000x + 54000y

There must be at least 360 deluxe cabins available so:

60x + 80y ≥ 360

There must be at least 680 standard cabins available so:

160x + 120y ≥ 680

And of course we cannot have negative numbers of ships so:

x ≥ 0
y ≥ 0

And the system is:

Minimize the objective function:
C = 44000x + 54000y
Under the constraints:
60x + 80y ≥ 360
160x + 120y ≥ 680
x ≥ 0
y ≥ 0

So we need to graph the constraint lines.
NOTE: Most instructors at A&M will have you shade the FALSE side.

For the line 60x + 80y = 360:

Let x = 0, and solve for y,
So we get 80y = 360 → y = 9/2
Thus the first point on the line is (0,9/2)
Let y = 0, solve for x,
And we get 60x = 360 → x = 6
Thus the second point on the line is (6,0)
Draw the line through (0, 9/2) and (6, 0)
Pick the point (0, 0) and put it into 60x + 80 y ≥ 360
→ 0 + 0 ≥ 360 is false so the true side is above the line.

For the line 160x + 120y = 680:

Let x = 0, and solve for y,
So we get 120y = 680 → y = 17/3
So our first point on this line is (0, 17/3)
Let y = 0, solve for x,
And we get 160x = 360 → x = 9/4
So our second point on the line is (9/4, 0)
Draw the line through the points (0, 17/3) and (9/4, 0)

Pick the point (0, 0) $160x + 120y \geq 680$
→ $0 \geq 680$ is false so the true side is above the line

For $x \geq 0$ the true side is everything to the right of the y-axis.

For $y \geq 0$ the true side is everything above the x-axis.

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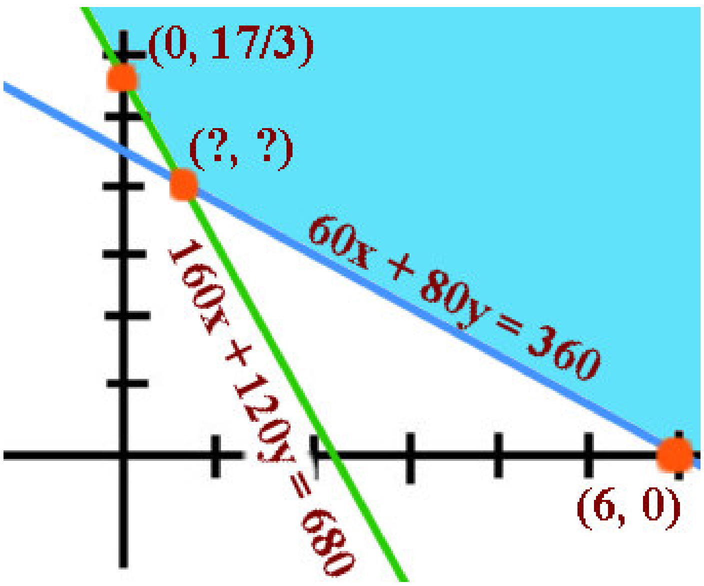


You now need to determine the intersection point of the lines:
Eq 1: $60x + 80y = 360$
Eq 2: $160x + 120y = 680$

Solve for y in Eq 2 and you get:
$160x + 120y = 680$ → $120y = 680 - 160x$
→ $y = 17/3 - (4/3)x$

Substitute $17/3 - (4/3)x$ in for y into Eq 1 and solve for x:
$60x + 80y = 360$ → $60x + 80*(17/3 - (4/3)x) = 360$
→ $60x + 1360/3 - (320/3)x = 360$
→ $(-140/3)x = -280/3$
→ $x = 2$

Put 2 in for x into Eq 2 and solve for y:
$160x + 120y = 680$ → $160*2 + 120y = 680$
→ $320 + 120y = 680$
→ $120y = 360$
→ $y = 3$

So the unknown 'corner' is actually (2, 3).   *Note our graph is not to scale above*

Now all we need to do is put all our corner points into our objective function:
C = 44000x + 54000y

We will use a table for this:

Point	Value of C = 44000x + 54000y
(0, 17/3)	306000
(6, 0)	264000
(2, 3)	250000

Recall we want to MINIMIZE cost
So we pick point (2, 3) which puts the cost at $250,000

So our answer is:

Use 2 type-A vessels and 3 type-B vessels for a minimal cost of $250,000

Problem 45:
TMA manufacturers 19 inch color television picture tubes in two separate locations, location I and location II. The output at location I is at most 6000 tubes/month, whereas the output at location II is at most 5000 tubes/month. TMA is the main supplier of picture tubes to Pulsar Corporation, its holding company, which has priority in having all its requirements met. In a certain month, Pulsar placed orders for 3000 and 4000 picture tubes to be shipped to two of its factories located in city A and city B, respectively. The shipping costs (in dollars) per picture tube from the two TMA plants to the Pulsar factories are as follows:

	To Pulsar Factories in City A	To Pulsar Factories in City B
From TMA Location I	$3	$2
From TMA Location II	$4	$5

Find a shipping schedule that meets the requirements of both companies while keeping costs to a minimum.

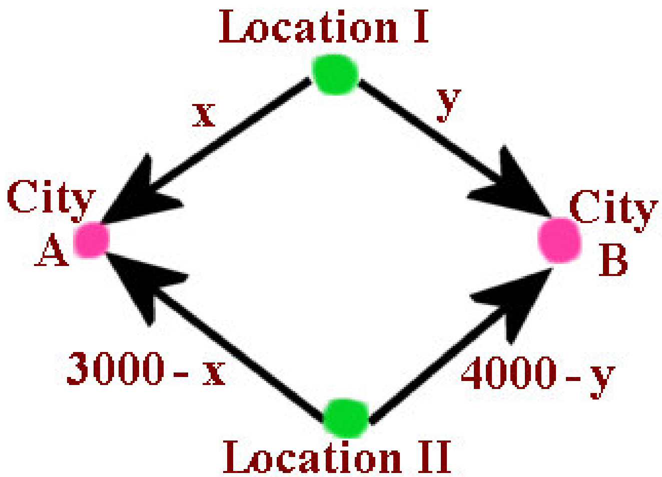
For this a picture showing how stuff moves will help.

Let x = the number of tubes sent from Location I to City A.

Note that city A needs 3000 tubes.
So the number of tubes sent from Location II to City A must be 3000- x.
(i.e. the number shipped from Location I and Location II to city A must add up to 3000)

Let y = the number of tubes sent from Location I to city B.
Note that city needs 4000 tubes.
So the number of tubes sent from Location II to City B must be 4000 – y.

The picture looks like this:



Our objective function to MINIMIZE is cost.
The cost of tubes leaving Location I = 3x + 2y
The cost of tubes leaving Location II = 4*(3000 – x) + 5*(4000 – y)
= 32000 – 4x – 5y
So our total cost is:

$C = (3x + 2y) + (32000 - 4x - 5y)$

$\mathbf{C = 32000 - x - 3y}$

Since Location I can only produce 6000 tubes per month we have the constraint:

$\mathbf{x + y \leq 6000}$

Since Location II can only produce 5000 tubes per month we also have:

$(3000 - x) + (4000 - y) \leq 5000$, or simplifying

$7000 - x - y \leq 5000$

$2000 \leq x + y$

$\mathbf{x + y \geq 2000}$

We also know that we cannot ship negative amounts of tubes so we know:

$\mathbf{x \geq 0}$

$\mathbf{y \geq 0}$

$3000 - x \geq 0 \rightarrow 3000 \geq x \rightarrow \mathbf{x \leq 3000}$

$4000 - y \geq 0 \rightarrow 4000 \geq y \rightarrow \mathbf{y \leq 4000}$

And so the system we are to solve is:

Minimize the objective function:

$C = 32000 - x - 3y$

Subject to the constraints:

$x + y \leq 6000$

$x + y \geq 2000$

$x \geq 0$

$y \geq 0$

$x \leq 3000$

$y \leq 4000$

So we need to graph the constraint lines.

NOTE: Most instructors at A&M will have you shade the FALSE side.

For the line $x + y = 6000$:

Let $x = 0$, and solve for y → $y = 6000$.
Thus the first point on the line is (0, 6000)
Let $y = 0$, solve for x → $x = 6000$
Thus the second point on the line is (6000,0)
Draw the line through (0, 6000) and (6000, 0)
Pick the point (0, 0) and put it into $x + y \leq 6000$
→ $0 + 0 \leq 6000$ is true so the true side is below the line.

For the line $x + y = 2000$:

Let $x = 0$, and solve for y → $y = 2000$.
Thus the first point on the line is (0, 2000)
Let $y = 0$, solve for x → $x = 2000$
Thus the second point on the line is (2000,0)
Draw the line through (0, 2000) and (2000, 0)
Pick the point (0, 0) and put it into $x + y \geq 2000$
→ $0 + 0 \geq 2000$ is false so the true side is above the line.

For $x \geq 0$ the true side is everything to the right of the y-axis.

For $y \geq 0$ the true side is everything above the x-axis.

For $x \leq 3000$ the true side is everything left of the vertical line $x = 3000$.

For $y \leq 4000$ the true side is everything below the horizontal line $y = 4000$.

NOTE:
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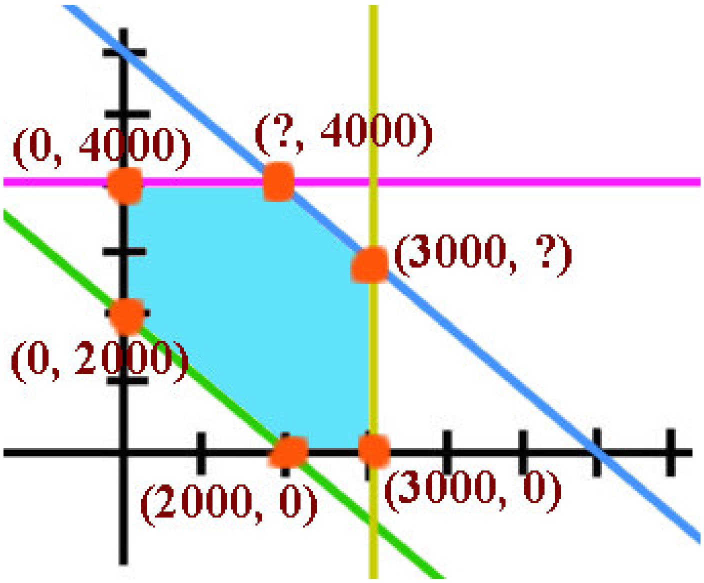
Each hash mark represents 1000



Now we must determine the intersection point of the lines:

Eq 1: x = 3000

Eq 2: x + y = 6000

Which is easy – just sub 3000 in for x into Eq 2 → y = 3000
And we arrive discover one unknown corner is (3000, 3000)

And now we must determine the intersection point of the lines:

Eq 3: y = 4000

Eq 4: x + y = 6000

Which is easy – just sub 4000 in for y into Eq 4 → x = 2000
And we arrive discover the other unknown corner is (2000, 4000)

Now all we need to do is put all our corner points into our objective function:

C = 32000 – x – 3y

We will use a table for this:

Point	**Value of C = 32000 – x – 3y**
(0,2000)	26000
(0, 4000)	20000
(2000, 4000)	18000
(3000, 3000)	20000
(3000, 0)	29000
(2000, 0)	30000

Recall we want to MINIMIZE cost
So we pick point (2000, 4000) which puts the cost at $18000

So our answer is:
Send 2000 tubes from Location I to City A.
Send 1000 tubes from Location II to City A

Send 4000 tubes from Location I to city B.
Send 0 tubes sent from Location II to City B.