Finite Math Section 4_1
Solutions and Hints

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for the book:

<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 4.1:

Learn to do these by hand. There are some programs for the TI-83 which may be useful. You will also find Excel extremely useful for solving these, and there is probably a helper utility available that is Java based and will run on a Web Browser.

**Notice in most of the solutions below I will use s1 instead of u,
s2 instead of v, and s3 instead of w.**

Problem 11:

Solve the linear programming problem by the simplex method:

Maximize $P = 3x + 4y$
subject to $x + y \le 4$
$2x + y \le 5$
$x \ge 0, y \ge 0$

Below is the Initial Tableau.
Tableau #1

x	y	s1	s2	p	
1	1	1	0	0	4
2	1	0	1	0	5
-3	-4	0	0	1	0

We must determine the pivot element. This element is found by first locating the most negative entry in the last row (-4) which is found in column 2. So our pivot column is column 2 (the y column).

We then had to find the pivot row. Below is the tableau with the ratios (c / a) for column 2 shown and the pivot column is highlighted:

x	y	s1	s2	p		ratio c/a
1	1	1	0	0	4	*4 / 1*
2	1	0	1	0	5	*5 / 1*
-3	-4	0	0	1	0	

We pick the row with the lowest ratio. In this case 4 < 5 so we find row 1 to be our pivot row.

Thus our pivot element is in row 1 and column 2. As highlighted below:
Tableau #1

x	y	s1	s2	p	
1	1	1	0	0	4
2	1	0	1	0	5
-3	-4	0	0	1	0

We now use row operations to 'zero-out' all entries above and below our pivot element (R2 – 2*R1 and R3 + 4*R1). The result you get should be:

Tableau #2

x	y	s1	s2	p	
1	1	1	0	0	4
1	0	-1	1	0	1
1	0	4	0	1	16

From this we see that:
x = 0, y = 4, s1 = 0, s2 = 1 and P = 16

So our optimal Solution: p = 16 with x = 0, y = 4

<u>Problem 12</u>:
Solve the linear programming problem by the simplex method:

Maximize $\quad P = 5x + 3y$

subject to $\quad x + y \leq 80$

$3x \leq 90$

$x \geq 0,\ y \geq 0$

Below is the Initial Tableau.

Tableau #1

x	y	s1	s2	p	
1	1	1	0	0	80
3	0	0	1	0	90
-5	-3	0	0	1	0

We need to find the pivot element. First we see –5 is the most negative number in the last row, so column 1 is our pivot column. We then establish the ratios c / a for this column to be:

x	y	s1	s2	p		*ratios*
1	1	1	0	0	80	*80 / 1*
3	0	0	1	0	90	*90 / 3= 30*
-5	-3	0	0	1	0	

And we see that row 2 has the least ratio, so it is the pivot row.

Thus our pivot element is in row 2 and column 1. So we use row operations to 'zero-out' everything above and below our pivot element (R2 = (1/3)R2, then R1- R2 and then R3 + 5R2). And we get the result of:

Tableau #2

x	y	s1	s2	p		
0	1	1	-1/3	0	50	
1	0	0	1/3	0	30	
0	-3	0	5/3	1	150	

Noticing we still have a negative number in the bottom row we will need to find another pivot element. As the 'most' negative number (only remaining) is in column 2, we know our pivot column is 2. We now need to find the pivot row by again taking ratios this time for column 2:

x	y	s1	s2	p		*ratios*
0	1	1	-1/3	0	50	*50 / 1*
1	0	0	1/3	0	30	*0 is not positive so ignore this row*
0	-3	0	5/3	1	150	

And we see our pivot row is row 1. We then zero out everything above and below our pivot element using row operations. Specifically R3 = R3 + 3R1. And we obtain the below:

Tableau #3

x	y	s1	s2	p	
0	1	1	-1/3	0	50
1	0	0	1/3	0	30
0	0	3	2/3	1	300

So we see that:
x = 30, y = 50, s1 = 0, s2 = 0, p = 300

And the optimal solution is: p = 300; x = 30, y = 50

Problem 13:
Solve the linear programming problem by the simplex method:
Maximize P = 10x + 12y subject to
$x + 2y \leq 12$
$3x + 2y \leq 24$
$x \geq 0, y \geq 0$

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	s1	s2	p	
1	2	1	0	0	12
3	2	0	1	0	24
-10	-12	0	0	1	0

Tableau #2

x	y	s1	s2	p	
1/2	1	1/2	0	0	6
2	0	-1	1	0	12
-4	0	6	0	1	72

Tableau #3

x	y	s1	s2	p	
0	1	3/4	-1/4	0	3
1	0	-1/2	1/2	0	6
0	0	4	2	1	96

So we see: x= 6, y = 3, s1 = 0, s2 = 0, p = 96

And the Optimal Solution: p = 96; x = 6, y =3

Problem 14:
Solve the linear programming problem by the simplex method:
Maximize P = 5x + 4y subject to
3x + 5y ≤ 12
4x + y ≤ 24
x ≥ 0, y ≥ 0

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	s1	s2	p	
3	5	1	0	0	12
4	1	0	1	0	24
-5	-4	0	0	1	0

Tableau #2

x	y	s1	s2	p	
1	5/3	1/3	0	0	4
0	-17/3	-4/3	1	0	8
0	13/3	5/3	0	1	20

And we see: x = 4, y = 0, s1 = 0, s2 = 8, p = 20

So the Optimal Solution is: p = 20; x = 4, y = 0

Problem 15:
Solve the linear programming problem by the simplex method:
Maximize P = 4x + 6y subject to
3x + y ≤ 24
2x + y ≤ 18
x + 3y ≤ 24
x ≥ 0, y ≥ 0

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	s1	s2	s3	p	
3	1	1	0	0	0	24
2	1	0	1	0	0	18
1	3	0	0	1	0	24
-4	-6	0	0	0	1	0

Tableau #2

x	y	s1	s2	s3	p	
8/3	0	1	0	-1/3	0	16
5/3	0	0	1	-1/3	0	10
1/3	1	0	0	1/3	0	8
-2	0	0	0	2	1	48

Tableau #3

x	y	s1	s2	s3	p	
0	0	1	-8/5	1/5	0	0
1	0	0	3/5	-1/5	0	6
0	1	0	-1/5	2/5	0	6
0	0	0	6/5	8/5	1	60

And we arrive at: x = 6, y = 6, s1 = 0, s2 = 0, s3 = 0, p = 60

Thus the Optimal Solution is: p = 60; x = 6, y = 6

Problem 18:
Solve the linear programming problem by the simplex method:
Maximize P = 3x + 3y + 4z subject to
$x + y + 3z \leq 15$
$4x + 4y + 3z \leq 65$
$x \geq 0, y \geq 0, z \geq 0$

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	z	s1	s2	p	
1	1	3	1	0	0	15
4	4	3	0	1	0	65
-3	-3	-4	0	0	1	0

Tableau #2

x	y	z	s1	s2	p	
1/3	1/3	1	1/3	0	0	5
3	3	0	-1	1	0	50
-5/3	-5/3	0	4/3	0	1	20

You could pick either row 1, column1 as the pivot element OR
you could pick row 1, column 2. Either choice will give you the same result.

Tableau #3

x	y	z	s1	s2	p	
1	1	3	1	0	0	15
0	0	-9	-4	1	0	5
0	0	5	3	0	1	45

And we get x = 15, y = 15, z = 0, s1 = 0, s2 = 5, p = 45.

So the Optimal Solution is:
p = 45; x = 15, y = 15, z = 0

Problem 19:
Solve the linear programming problem by the simplex method:
Maximize P = 3x + 4y + z subject to
$3x + 10y + 5z \le 120$
$5x + 2y + 8z \le 6$
$8x + 10y + 3z \le 105$
$x \ge 0, y \ge 0, z \ge 0$

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	z	s1	s2	s3	p	
3	10	5	1	0	0	0	120
5	2	8	0	1	0	0	6
8	10	3	0	0	1	0	105
-3	-4	-1	0	0	0	1	0

Tableau #2

x	y	z	s1	s2	s3	p	
-22	0	-35	1	-5	0	0	90
5/2	1	4	0	1/2	0	0	3
-17	0	-37	0	-5	1	0	75
7	0	15	0	2	0	1	12

And we see that:
x = 0, y = 3, z = 0, s1 = 90, s2 = 0, s3 = 75, p = 12

So the Optimal Solution is:
p = 12; x = 0, y = 3, z = 0

Problem 20:
Solve the linear programming problem by the simplex method:
Maximize P = x + 2y - z subject to
$2x + y + z \leq 14$
$4x + 2y + 3z \leq 28$
$2x + 5y + 5z \leq 30$
$x \geq 0, y \geq 0, z \geq 0$

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	z	s1	s2	s3	p	
2	1	1	1	0	0	0	14
4	2	3	0	1	0	0	28
2	5	5	0	0	1	0	30
-1	-2	1	0	0	0	1	0

Tableau #2

x	y	z	s1	s2	s3	p	
8/5	0	0	1	0	-1/5	0	8
16/5	0	1	0	1	-2/5	0	16
2/5	1	1	0	0	1/5	0	6
-1/5	0	3	0	0	2/5	1	12

Tableau #3

x	y	z	s1	s2	s3	p	
0	0	-1/2	1	-1/2	0	0	0
1	0	5/16	0	5/16	-1/8	0	5
0	1	7/8	0	-1/8	1/4	0	4
0	0	49/16	0	1/16	3/8	1	13

And so: x = 5, y = 4, z = 0, s1 = 0, s2 = 0, s3 = 0, p =13

Thus the Optimal Solution is:
p = 13; x = 5, y = 4, z = 0

Problem 24:
Solve the linear programming problem by the simplex method:
Maximize P = 2x + 6y + 6z subject to
$2x + y + 3z \leq 10$
$4x + y + 2z \leq 56$
$6x + 4y + 3z \leq 126$
$2x + y + z \leq 32$
$x \geq 0, y \geq 0, z \geq 0$

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	z	s1	s2	s3	s4	p	
2	1	3	1	0	0	0	0	10
4	1	2	0	1	0	0	0	56
6	4	3	0	0	1	0	0	126
2	1	1	0	0	0	1	0	32
-2	-6	-6	0	0	0	0	1	0

Tableau #2

x	y	z	s1	s2	s3	s4	p	
2	1	3	1	0	0	0	0	10
2	0	-1	-1	1	0	0	0	46
-2	0	-9	-4	0	1	0	0	86
0	0	-2	-1	0	0	1	0	22
10	0	12	6	0	0	0	1	60

And we arrive at:
x = 0, y = 10, z = 0, s1 = 0, s2 = 46, s3 = 86, s4 = 22, p = 60

So the Optimal Solution is:
p = 60; x = 0, y = 10, z = 0

Problem 30:
Kane Manufacturing has a division that produces two models of hibachis, model A and model B. To produce each model A hibachi requires 3lb of cast iron and 6 min of labor. To produce each model B hibachi requires 4 lb of cast iron and 3 min of labor. The profit for each model A hibachi is $2 and the profit for each model B hibachi is $1.50. If 1000 lb of cast iron and 20 labor hours are available for the production of hibachis each day, how many hibachis of each model should the division produce to maximize Kane's profits? What is the largest profit the company can realize? Is there any raw material left over?

Let x = quantity of model A hibachis produced.
Let y = quantity of model B hibachis produced.

Our objective function to maximize is profit: $P = 2x + 1.5y$

We have 1000 lb of cast iron available.
The amount of iron used in making model A's = 3x
The amount of iron used in making model B's = 4y
So constraint 1 is:
Con 1: $3x + 4y \le 1000$

We have 20 labor hours = 1200 minutes available.
The amount of time (in minutes) spent creating model A's = 6x
The amount of time (in minutes) spent creating model B's = 3y
So constraint 2 is:
Con 2: $6x + 3y \le 1200$

Thus we are to:
Maximize $P = 2x + 1.5y$ subject to
$3x + 4y \le 1000$
$6x + 3y \le 1200$

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	s1	s2	p	
3	4	1	0	0	1000
6	3	0	1	0	1200
-2	-3/2	0	0	1	0

Tableau #2

x	y	s1	s2	p	
0	5/2	1	-1/2	0	400
1	1/2	0	1/6	0	200
0	-1/2	0	1/3	1	400

Tableau #3

x	y	s1	s2	p	
0	1	2/5	-1/5	0	160
1	0	-1/5	4/15	0	120
0	0	1/5	7/30	1	480

So we read the solution:
x = 120, y = 160, s1 = 0, s2 = 0 and p = 480

We conclude that making 120 model A hibachis and 160 model B hibachis will maximize the profit to be $480.
We note that both slack variables are zero so there will be no resources left over.

Problem 32:
A financier plans to invest up to $500,000 in two projects. Project A yields a return of 10% on the investment, whereas project B yields a return of 15% on the investment. Because the investment in project B is riskier than the investment in project A, she has decided that the investment in project B should not exceed 40% of the total investment. How much should the financier invest in each project in order to maximize the return on her investment? What is the maximum return?

Let x = amount invested in project A
Let y = amount invested in project B

The total return of the investments is: $P = 0.1x + 0.15y$

There is at most \$50000 available.
So we get: $x + y \leq 500000$

The investment in B is not to exceed 40% of the total investment.
So $y \leq 0.40 * (x + y) \rightarrow y \leq 0.4x + 0.4y$
$\rightarrow -0.4x + 0.6y \leq 0$

Thus our goal is to:
Maximize $P = 0.1x + 0.15y$ subject to
$x + y \leq 500000$
$-0.4x + 0.6y \leq 0$

Below are the tableaus for each step. The pivot elements are highlighted. The row operations are left for you to determine (it should be easy – just work backwards if you need to):

Tableau #1

x	y	s1	s2	p	
1	1	1	0	0	500000
-2/5	3/5	0	1	0	0
-1/10	-3/20	0	0	1	0

Tableau #2

x	y	s1	s2	p	
5/3	0	1	-5/3	0	500000
-2/3	1	0	5/3	0	0
-1/5	0	0	1/4	1	0

Tableau #3

x	y	s1	s2	p	
1	0	3/5	-1	0	300000
0	1	2/5	1	0	200000
0	0	3/25	1/20	1	60000

And we see that: x = 300000, y = 200000, s1 = 0, s2 = 0, p = 60000

So we conclude that she will achieve a maximal gain of \$60,000 by investing \$300,000 in project A and \$200,000 in project B.