Finite Math Section 6_1
Solutions and Hints

by Brent M. Dingle

for the book:
Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 6.1:
Welcome to set theory. In this chapter you need to memorize some definitions of terms. It will make things much easier later. Most instructors assume you know all of this from high school and will not talk much about it (with some exceptions =).

Know what a **set** is.
Know what the **elements** of a set are.
Know what a **subset** is.
Know the difference between a subset and a **proper subset**.
Know what the **empty set** is and the notation for it.
Know what the **complement** of a set is, particularly when given a **universal set**.
Know **De Morgan's Laws**.

You will of course also need to be able to perform **set operations**, but you should be able to pick that up by doing problems.

Likewise you will also need to know what a **Venn diagram** is, what **set-builder notation** is and what **roster notation** is; but those should come simply by doing the problems.

Problem 18:
Let A = { a, e, l, t, r }. Which of the following sets are equal to A?
  a. { x | x is a letter of the word *later* }
  b. { x | x is a letter of the word *latter* }
  c. { x | x is a letter of the word *relate* }

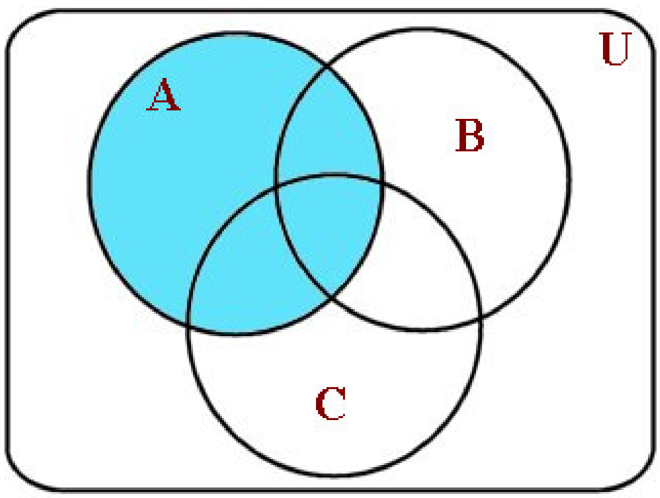
All the stated sets are equal to A.

The word later is made up of the letters: l, a, t, e, r which is all the letters in A.
The word latter is also made up of the letter: l, a, t, e, r again which is all the letter of A.
The word relate is made up of the letters: r, e, l, a, t again which is all the letters of A.

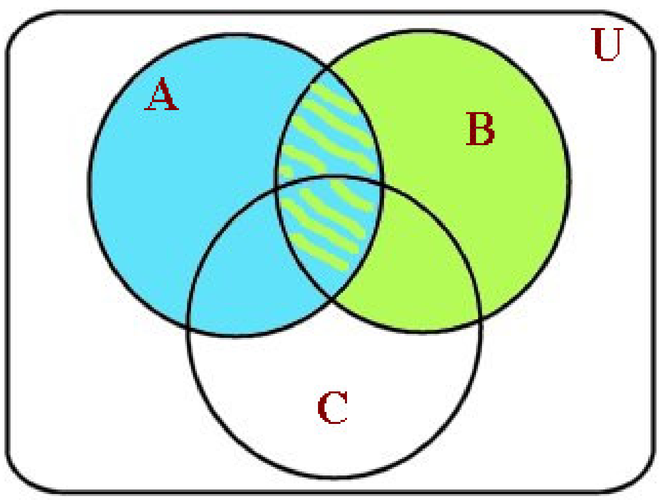
Problem 30:
a. Shade the portion of the figure that represents $A \cap B \cap C^C$
b. Shade the portion of the figure that represents $A^C \cap B \cap C$

Recall intersection is where stuff overlaps.

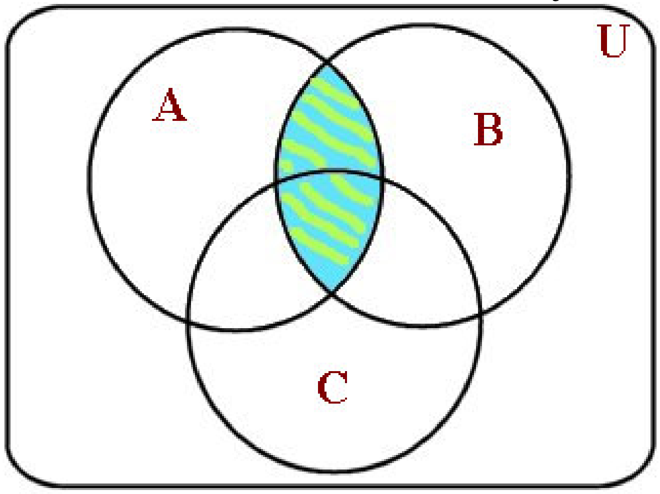
30 a: For $A \cap B \cap C^C$:
First shade A:



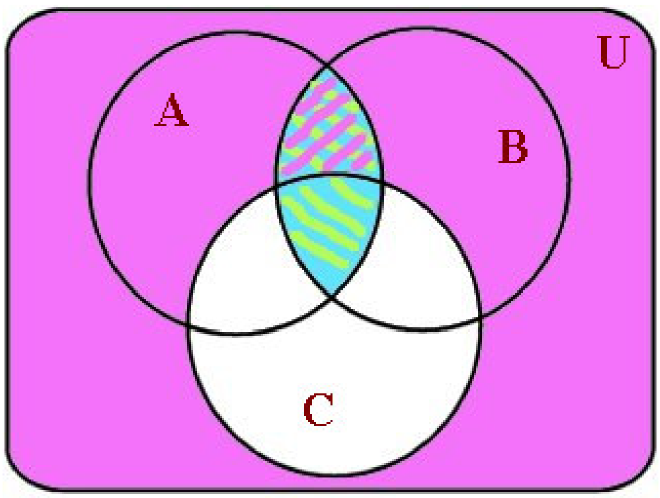
Now shade B:



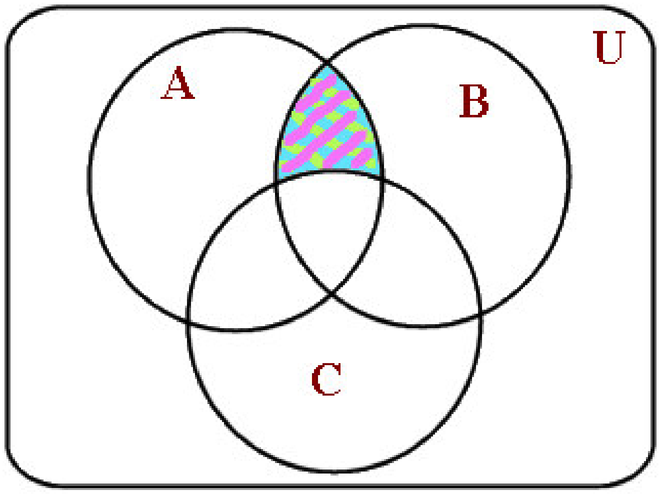
Notice the intersection, or rather $A \cap B$, is just:



Now shade C^C:

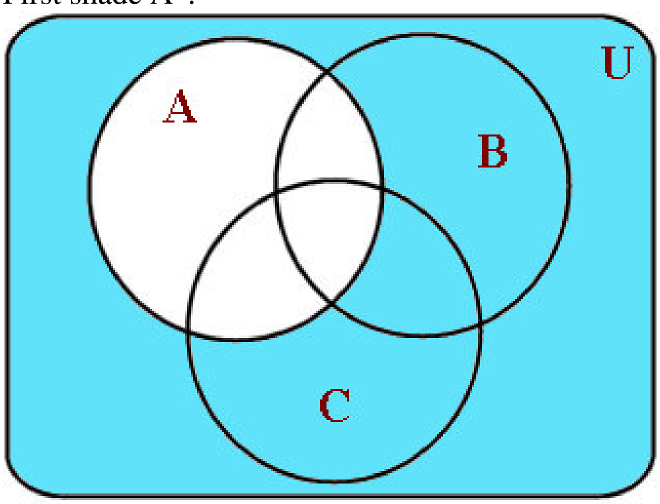


Notice it intersects $A \cap B$ like this (which gives us our answer):

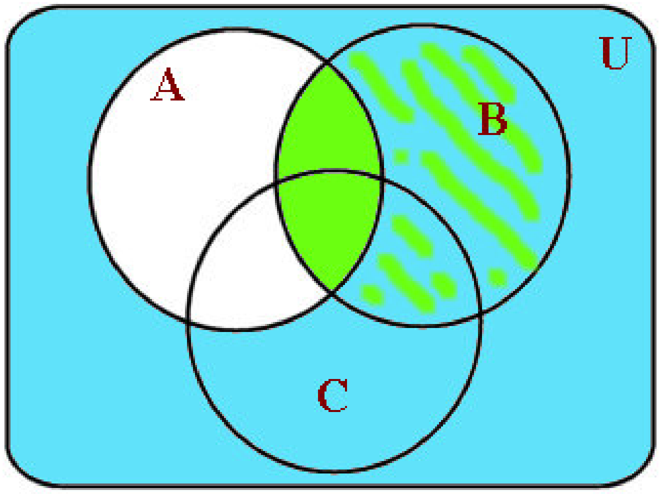


The above is our answer for $A \cap B \cap C^C$.

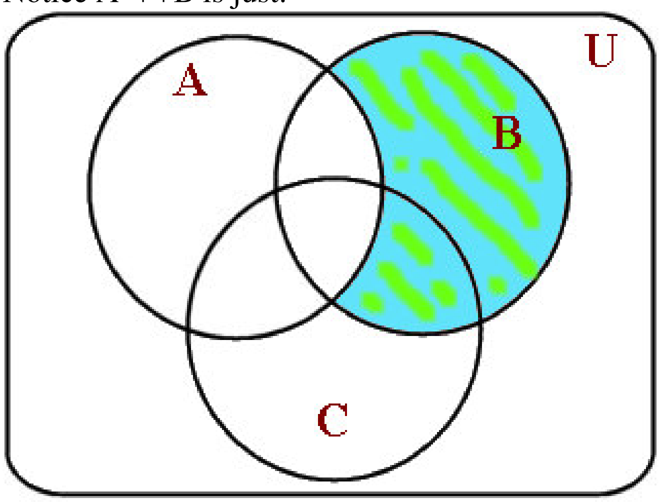
30 b: For $A^C \cap B \cap C$:
First shade A^C:



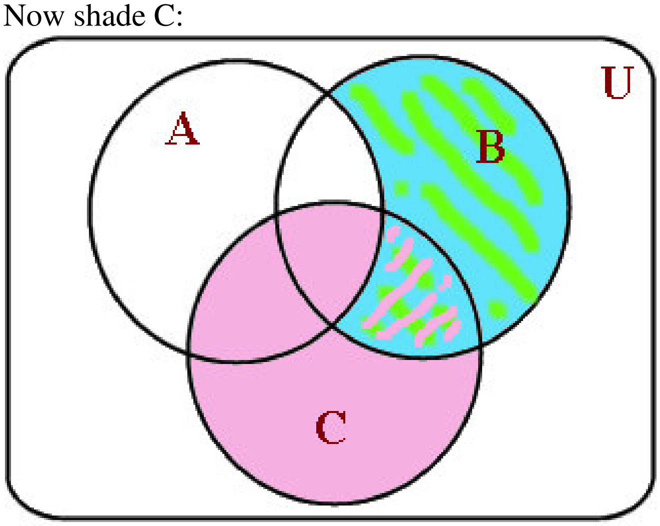
Then shade B:



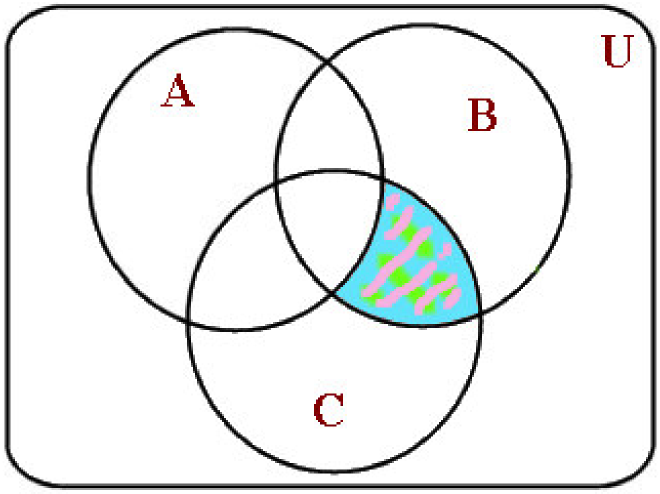
Notice $A^C \cap B$ is just:



Now shade C:



And we have C intersect ($A^C \cap B$) as below:



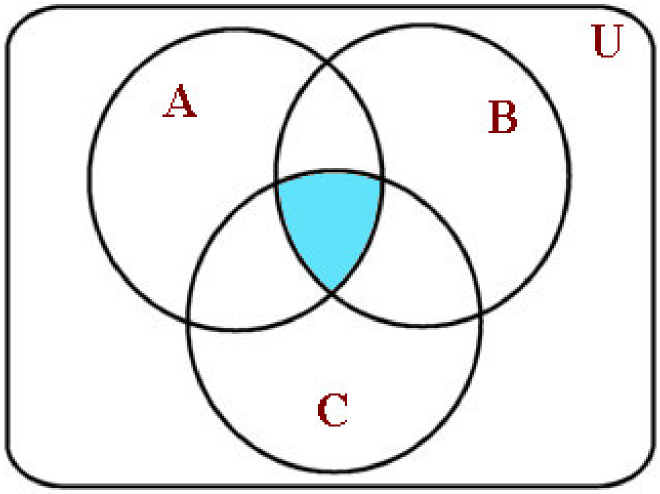
Our answer for $A^C \cap B \cap C$ is above.

<u>Problem 32</u>:
a. Shade the portion of the figure that represents $A \cap (B \cap C)^C$.
b. Shade the portion of the figure that represents $(A \cap B \cap C)^C$

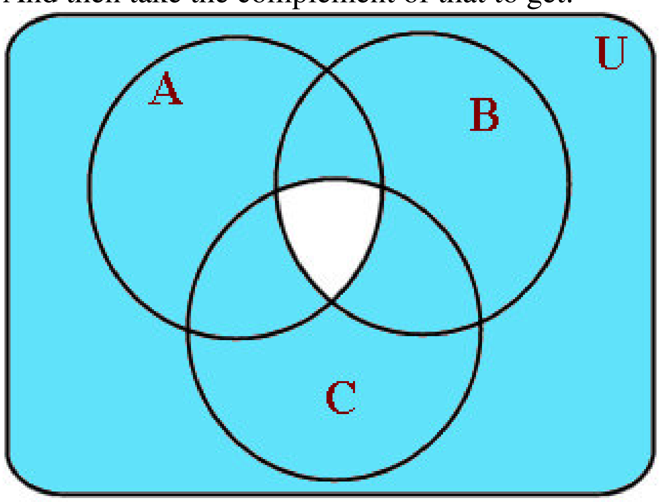
<u>For part 32 (a)</u>
Recall DeMorgan's law: $(X \cap Y)^C = X^C \cap Y^C$
So $A \cap (B \cap C)^C$ becomes $A \cap B^C \cap C^C$.

I'll leave the shading for you to do.
Also understand you don't 'have to' apply DeMorgan's law – you can solve it other ways, but it's a good thing to get some practice doing.

<u>For part 32 (b)</u>
There is no rule for what $(A \cap B \cap C)^C$ becomes.
So notice that $A \cap B \cap C$ is just:



And then take the complement of that to get:



The above is the answer for: $(A \cap B \cap C)^C$.

<u>Problem 34</u>:
U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }
A = { 1, 3, 5, 7, 9 }
B = { 2, 4, 6, 8, 10 }
C = { 1, 2, 4, 5, 8, 9 }

<u>34 a. Find $C \cap C^C$.</u>
First determine C^C = { 3, 6, 7, 10 } = { everything in U and NOT in C }
So

$C \cap C^C = \{ 1, 2, 4, 5, 8, 9 \} \cap \{ 3, 6, 7, 10 \}$ *notice there is no overlap*

$$\mathbf{C \cap C^C = \varnothing}$$

34 b. Find $(A \cap C)^C$.

You could use DeMorgan's law on this. I'm not going to.

First find $(A \cap C)$:

$A \cap C = \{ 1, 3, 5, 7, 9 \} \cap \{ 1, 2, 4, 5, 8, 9 \}$ *overlaps on 1, 5 and 9*

$A \cap C = \{ 1, 5, 9 \}$

Now find everything in U that is NOT in $A \cap C$

$$\mathbf{(A \cap C)^C = \{ 2, 3, 4, 6, 7, 8, 10 \}}$$

34 c. Find $A \cup (B \cap C)$.

First find $B \cap C$:

$B \cap C = \{ 2, 4, 6, 8, 10 \} \cap \{ 1, 2, 4, 5, 8, 9 \}$ *overlaps on 2, 4 and 8*

$B \cap C = \{ 2, 4, 8 \}$

Recall $A = \{ 1, 3, 5, 7, 9 \}$

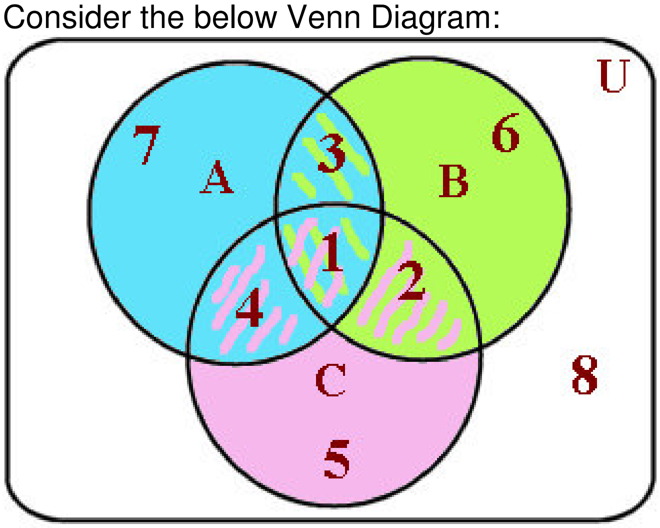
And $A \cup (B \cap C)$ is (everything in A) and (everything in $B \cap C$) without duplicates:

$A \cup (B \cap C) = \{ 1, 3, 5, 7, 9, 2, 4, 8 \}$ *and we will put it into a better order*

$$\mathbf{A \cup (B \cap C) = \{ 1, 2, 3, 4, 5, 7, 8, 9 \}}$$

Problem 52:

Consider the below Venn Diagram:



A = { x $\in$ U | x has taken the subway }
B = { x $\in$ U | x has taken a cab }
C = { x $\in$ U | x has taken a bus }

52 a. Express Region 3 in set notation and in words.

The set notation "reasoning"
Region 3 is in A and
Region 3 is in B and
Region 3 is not in C

So we get Region 3 = $A \cap B \cap C^c$

In words region 3 represents all tourists who have
taken the subway AND a cab AND NOT a bus

52 b. Express Region 4 and 6 together in set notation and in words.
The "reasoning"
Region 4 is in A and C and not in B.
Region 6 is in B and not A and not C.

So Region 4 = $A \cap C \cap B^C$.
Region 6 = $B \cap A^C \cap C^C$.

So if we take region 4 and 6 together (combine them, or, say union them) we get:

$$(A \cap C \cap B^C) \cup (B \cap A^C \cap C^C)$$

Notice there is no real good way to simplify that any further.

In words region 4 and 6 together represents all tourists who have
taken the subway AND a bus AND NOT a cab
OR
taken a cab AND not the subway AND NOT a bus

52 c. Express Regions 5, 6 and 7 together in set notation and in words.
The "reasoning:"
Region 5 is in C and not in A and not in B → $C \cap A^C \cap B^C$.

Region 6 is in B and not A and not C → $B \cap A^C \cap C^C$.
Region 7 is in A and not in B and not in C → $A \cap B^C \cap C^C$.

So putting them together:

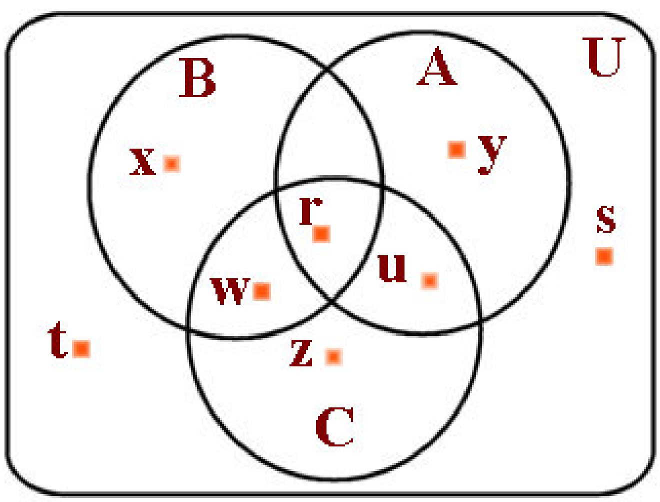
$$(C \cap A^C \cap B^C) \cup (B \cap A^C \cap C^C) \cup (A \cap B^C \cap C^C)$$

Which also does not simplify to anything nice.

However it words it can be stated well:

Region 5, 6 and 7 together represent all tourists who have used exactly one of the three transportation options.

Problem 62:
Consider the below Venn Diagram:



62 a. Find the points that belong to $A \cap (B \cup C)$:

First find $B \cup C$:
$B = \{ x, r, w \}$
$C = \{ r, w, u, z \}$
So
$B \cup C = \{ x, r, w, u, z \}$

Now find $A \cap (B \cup C)$

$A = \{ r, u, y, v \}$ *notice this overlaps $B \cup C$ on the points r and u*

So

$$\mathbf{A \cap (B \cup C) = \{ r, u \}}$$

<u>62 b. Find the points that belong to $(B \cap C)^C$:</u>

Notice $U = \{ s, y, u, z, w, r, v, x, t \}$

Notice $B \cap C = \{ w, r \}$

So $(B \cap C)^C$ is everything in U except for w and r

$$\mathbf{So\ (B \cap C)^C = \{ s, y, u, z, v, x, t \}}$$