Finite Math Section 6_2
Solutions and Hints

by Brent M. Dingle

for the book:

<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 6.2:

This section is pretty obvious, most instructors will not spend much time talking about it.
The important thing to note is the notation:

n(A) means number of elements in the set A
$n(A \cap B)$ means the number of elements in $A \cap B$
$n(A \cup B)$ means the number of elements in $A \cup B$
n(A) + n(B) means the number of elements in A plus the number in B

You may also wish to memorize the fact:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Notice $n(A \cup B)$ may not be equal to $n(A) + n(B)$.
For example let A = { a, b, c } and B = { b, c, d }
Then $n(A \cup B) = 4$, but $n(A) + n(B) = 6$

Problem 8:

If $n(A) = 10$, $n(A \cup B) = 15$ and $n(B) = 8$, what is $n(A \cap B)$?

This may be solved in several ways. The easiest is to remember the formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Then all you need to do is plug numbers in and solve:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ → $15 = 10 + 8 - n(A \cap B)$
→ $15 = 18 - n(A \cap B)$
→ $-3 = -n(A \cap B)$

$$\rightarrow \mathbf{3 = n(A \cap B)}$$

Another way to solve this problem is to think like this (perhaps using a picture):

The problem says: There are 10 things in A
The problem says: There are 8 things in B
Which gives at most 18 unique things in $A \cup B$
But the problem says: there are only 15 unique things in $A \cup B$
So there must be $18 - 15 = 3$ things in both A and B
So $n(A \cap B)$ is 3.

I would recommend memorizing the formula.

Problem 12:

If $n(B) = 6$, $n(A \cup B) = 14$ and $n(A \cap B) = 3$, find $n(A)$.

This may be solved in several ways. The easiest is to remember the formula:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

And plug stuff in:

$14 = n(A) + 6 - 3 \rightarrow 14 = n(A) + 3$

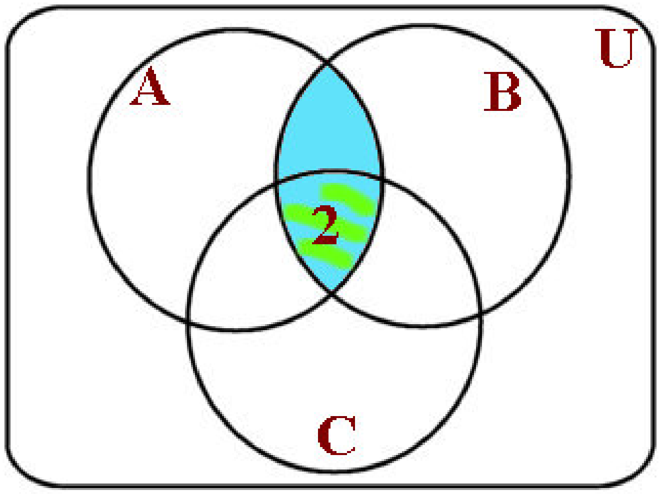
$$\rightarrow \mathbf{11 = n(A)}$$

Problem 15:

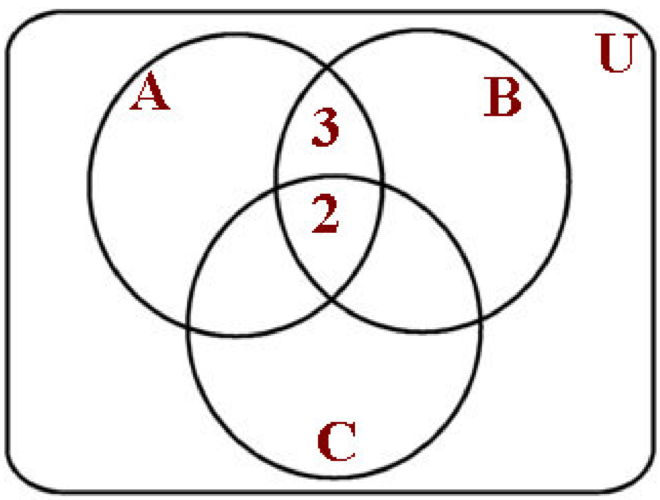
If $n(A) = 12$, $n(B) = 12$, $n(A \cap B) = 5$, $n(A \cap C) = 5$,
$n(B \cap C) = 4$, $n(A \cap B \cap C) = 2$, and $n(A \cup B \cup C) = 25$,
find $n(C)$.

For this a Venn Diagrams will help:

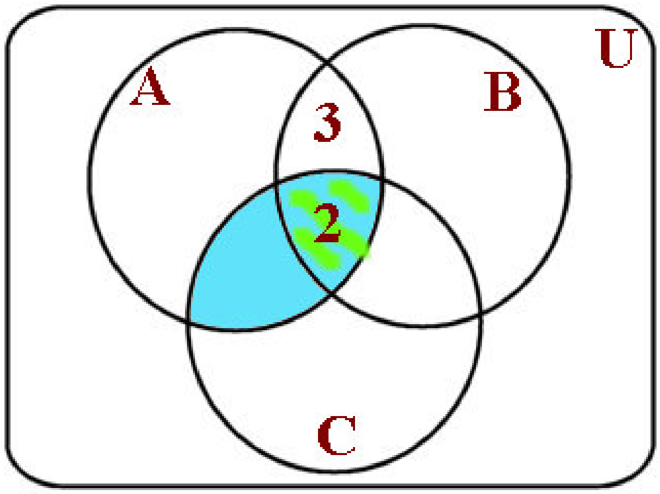
First we notice that $n(A \cap B \cap C) = 2$, AND we know $n(A \cap B) = 5$:



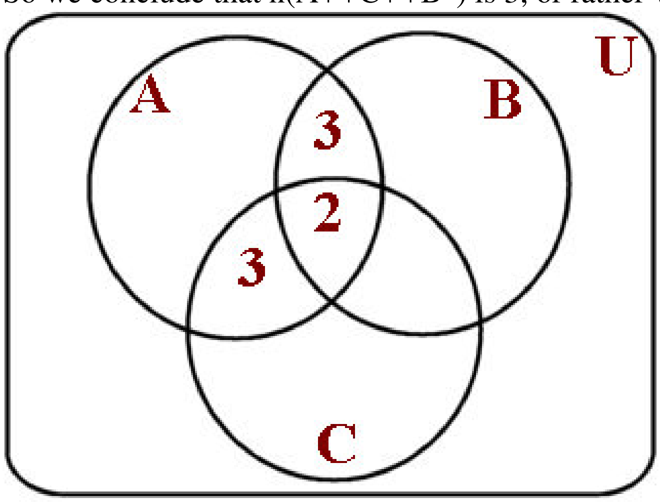
So we know the entire blue region = $n(A \cap B) = 5$.
We know the blue green region = $n(A \cap B \cap C) = 2$.
So we conclude that $n(A \cap B \cap C^C)$ is 3, or rather we get:



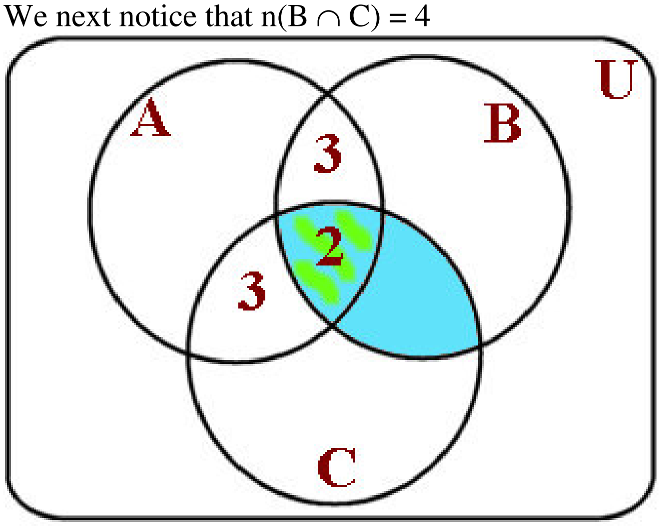
We next notice that $n(A \cap C) = 5$



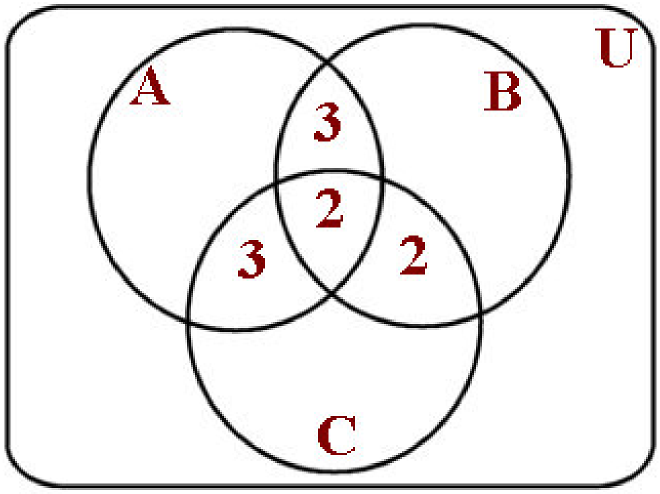
So again in the above picture the entire blue section = $n(A \cap C) = 5$.
We know that the blue green region = $n(A \cap B \cap C) = 2$
So we conclude that $n(A \cap C \cap B^C)$ is 3, or rather we get:



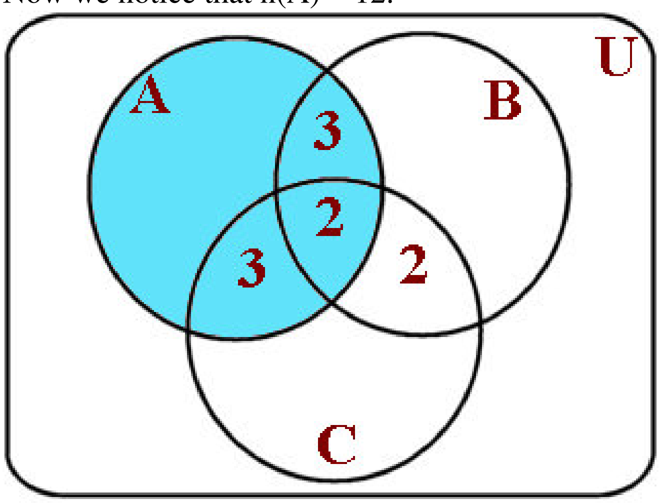
We next notice that $n(B \cap C) = 4$



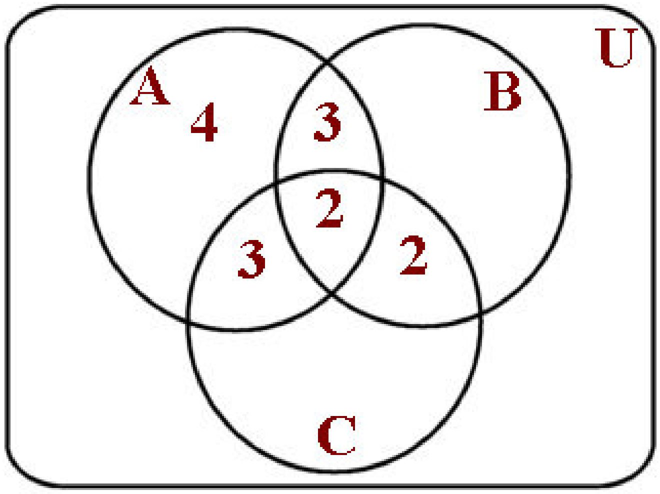
So in the above picture the entire blue section = $n(B \cap C) = 4$.
We know that the blue green region = $n(A \cap B \cap C) = 2$
So we conclude that $n(B \cap C \cap A^C)$ is 2, or rather we get:



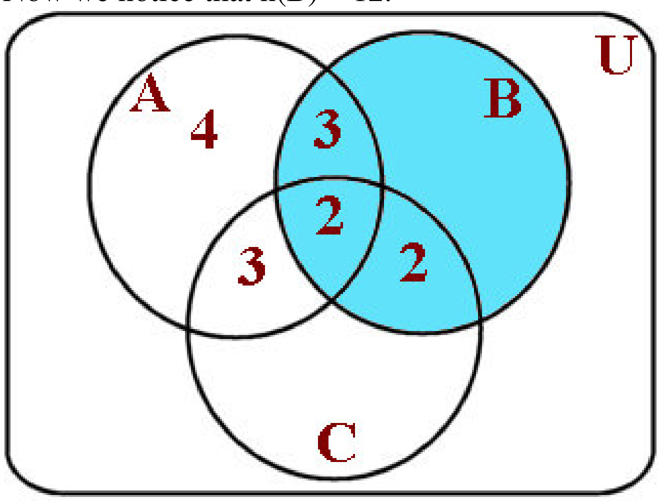
Now we notice that $n(A) = 12$.



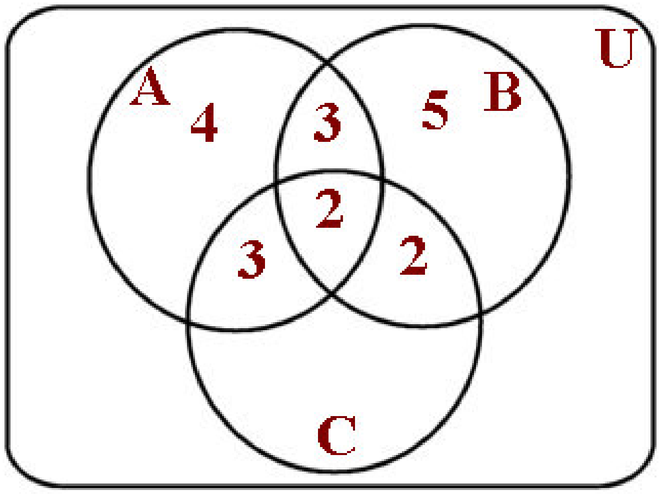
So in the above picture the entire blue section has 12 elements.
Currently we have $3 + 2 + 3 = 8$ already counted.
So there are 4 that we need to include.
Thus we arrive at:



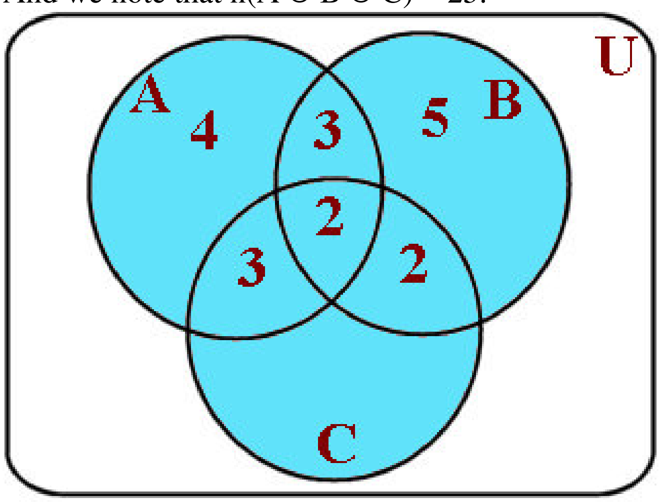
Now we notice that n(B) = 12.



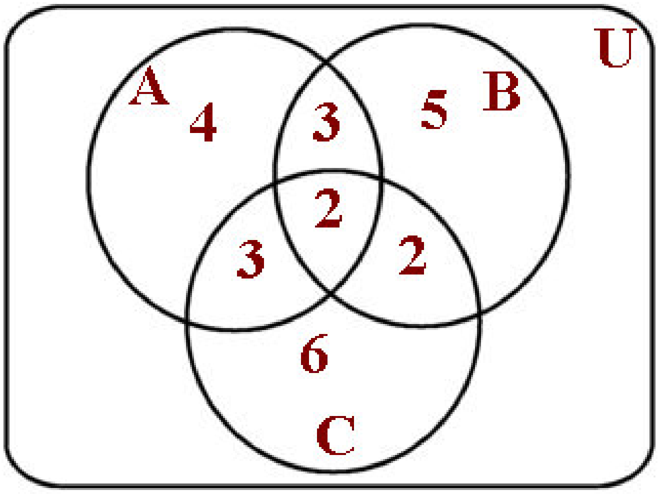
So in the above picture the blue region should have 12 elements total.
However we only have 3 + 2 + 2 = 7 accounted for.
So we need to add 5.
Thus we get:



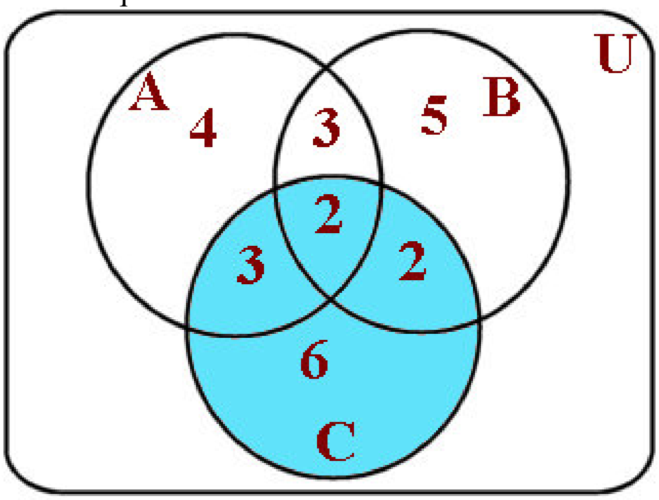
And we note that n(A ∪ B ∪ C) = 25.



So in the above picture the entire blue region should have 25 elements.
Yet it only has: 4 + 3 + 5 + 3 + 2 + 2 = 19.
So it needs 6 more added.
So we get:



And the question was to find the number of elements in C.



So we see that n(C) = the number of elements in the blue region above
which equals 3 + 2 + 2 + 6 = 13.

And we conclude:

n(C) = 13

<u>Problem 18</u>:
Of 100 clock radios sold recently in a department store, 70 had FM circuitry and 90 had AM circuitry. How many radios had both FM and AM circuitry? How many could receive FM transmissions only? How many could receive AM transmissions only?

Let A = { radios sold with FM circuitry (and maybe AM) }

Let B = { radios sold with AM circuitry (and maybe FM) }

$n(A \cup B) = 100$ = total sold.
$n(A) = 70$ = those sold with FM (and maybe AM)
$n(B) = 90$ = those sold with AM (and maybe FM_

From our favorite memorize equation we know:

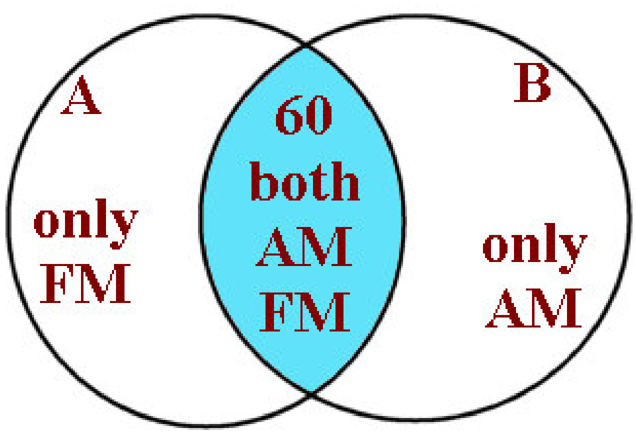
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

So plugging stuff in we get:

$$100 = 70 + 90 - n(A \cap B) \rightarrow 100 = 160 - n(A \cap B)$$
$$\rightarrow -60 = -n(A \cap B)$$
$$\rightarrow 60 = n(A \cap B)$$

So the number of radios with both FM and AM = $n(A \cap B) = 60$.

To see how many just had FM and how many just had AM consider the below Venn Diagram:



Since $n(A) = 70$, we see that $n(\{\text{only FM}\}) = 70 - n(A \cap B)$.

So number of clock radios with FM only = 70 – 60 = 10

Likewise since $n(B) = 90$, we see that $n(\{\text{ only AM}\}) = 90 - n(A \cap B)$

So number of clock radios with AM only = 90 – 60 = 30

Problem 22:
Of 50 employees of a store located in downtown Boston, 18 people take the subway to work, 12 take the bus and 7 take both the subway and the bus.

22a. How many employees take the subway OR the bus to work?
Let A = { people who take the subway }
Let B = { people who take the bus }

So
$n(A) = 18$
$n(B) = 12$
$n(A \cap B) = 7$

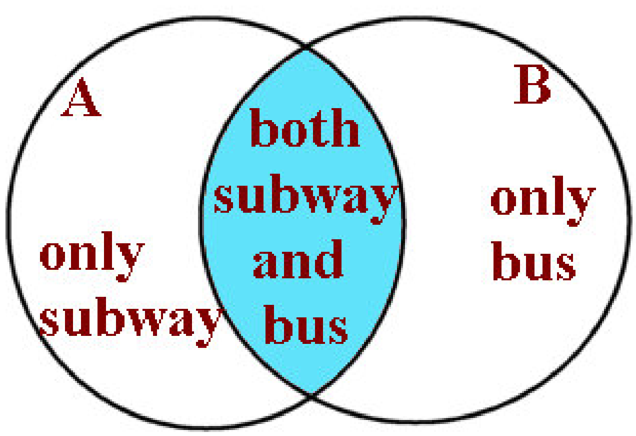
The number of employees who take the subway OR bus = $n(A \cup B)$.
Use our favorite equation to solve this:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

So plugging stuff in we get:

$$n(A \cup B) = 18 + 12 - 7 \quad \rightarrow \mathbf{n(A \cup B) = 23}$$

22b. How many employees take only the bus to work?
Consider the below Venn diagram:



Since the total number in B = n(B) = 12
and the number that take both the subway and the bus = $n(A \cap B) = 7$ (the blue section)
We see the region for bus only must have 12 – 7 = 5 people.
So we arrive at the conclusion:

The number of people who only take the bus = 5.

Similarly we notice the number of people who only take the subway must be:
$n(A) - n(A \cap B) = 18 - 7 = 11$.

22c. Take either the bus or the subway to work?
Notice "either the bus or the subway" means either the bus or the subway and NOT both.

From the work done in part 22b we know that 5 people only take the bus and 11 people only take the subway. So we arrive at the conclusion

The number of people taking either the bus or the subway (and not both) is 5 + 11 = 16 people.

22d. Get to work by some other means?
We are told there are a total of 50 people.
Since $n(A \cup B) = 23$, we know that only 23 of them rely on the subway and/or the bus.
Everyone else must get to work by some other means. So we find

50 – 23 = 27 people get to work by other means.

Problem 32:
Results of the Department of Education (DoEd) survey of SAT test scores in 22 states showed that:

- 10 states had an average composite test score of at least 1000 during the past 3 years.
- 15 states had an increase of at least 10 points in the average composites test score during the past 3 years.
- 8 states had both an average composite test score of at least 1000 and an increase in the average composite test score of at least 10 points during the past 3 years.

32a. How many of the 22 states had composite scores less than 1000 and showed an increase of at least 10 points over the last 3 years?

Let A = { states that had an average score of at least 1000}
Let B = { states that had an increase of at least 10 points }

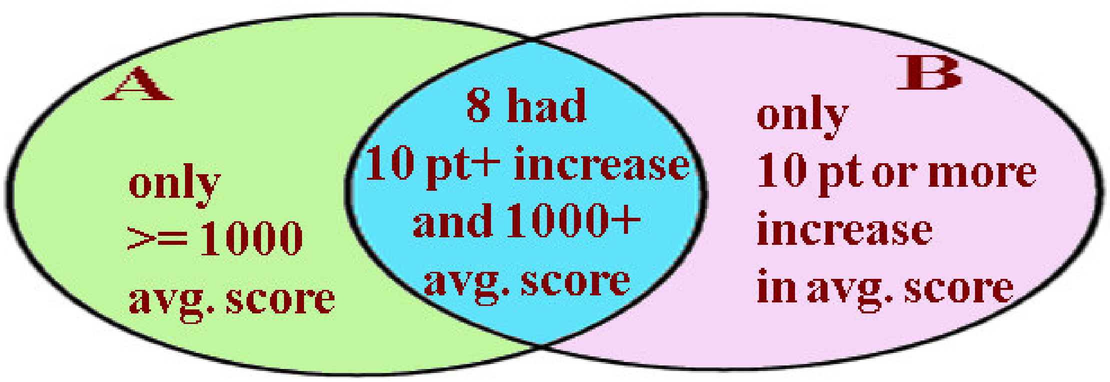
Aside: Notice that $n(A \cup B)$ does NOT equal 22 – some states had scores less than 1000 and did NOT have an increase of 10 points in their score.

We are given:
$n(A) = 10$
$n(B) = 15$

We are told that 8 states experienced both conditions.
So $n(A \cap B) = 8$.

And we rephrase the question to:
How many states had ONLY an increase of 10 pts or more?

Now consider the below Venn diagram:



We know $n(B) = 15$ and $n(A \cap B) = 8$.
We need to find the number of states in the pinkish region.
We see that B = pink + blue region.
So we say

n(B) = n({only had 10 point or more increase in avg. score} + 8
15 = n({only had 10 point or more increase in avg. score} + 8
7 = n({only had 10 point or more increase in avg. score}

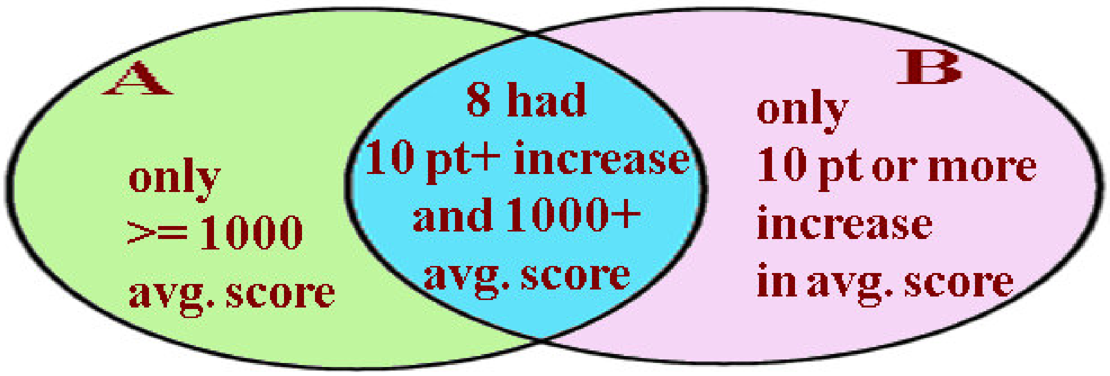
And we conclude:

7 states
had composite scores less than 1000 and showed an increase of at least 10 points over the last 3 years

32b. How many of the 22 states had composite scores of at least 1000 and did NOT show an increase of at least 10 points over the last 3 years?

This problem works the same as part (a).
First rephrase the question to be:
How many states ONLY had a score of at least 1000?

And we again consider the Venn diagram:



We know $n(A) = 10$ and $n(A \cap B) = 8$.
We need to find the number of states in the green region.
We see that A = green region + blue region.
So we say

$n(A) = n(\{\text{only had a score of at least } 1000\}) + n(A \cap B)$

$10 = n(\{\text{only had a score of at least } 1000\}) + 8$

$2 = n(\{\text{only had a score of at least } 1000\})$

And we conclude

2 states
had composite scores of at least 1000 and did NOT show an increase of at least 10 points over the last 3 years

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Problem 34:
To plan the number of meals to be prepared in a college cafeteria, a survey was conducted, and the following data was obtained:

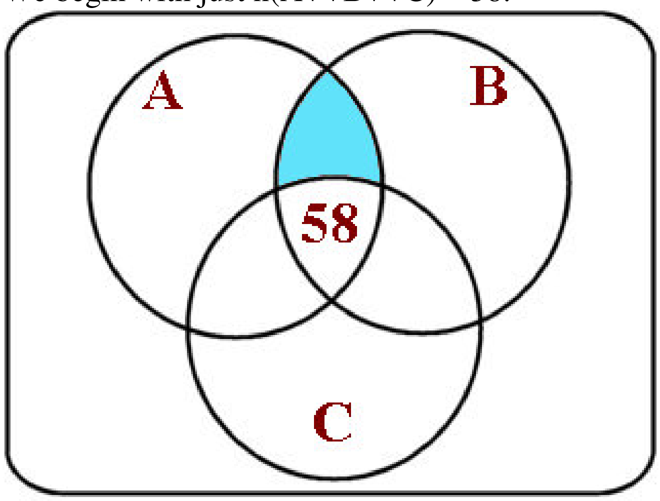
- 130 students ate breakfast
- 180 students ate lunch

- 275 students ate dinner
- 68 students ate breakfast AND lunch
- 112 students ate breakfast AND dinner
- 90 students ate lunch AND dinner
- 58 students ate all three meals (breakfast AND lunch AND dinner)

Before doing anything let's make a Venn diagram of the above data:

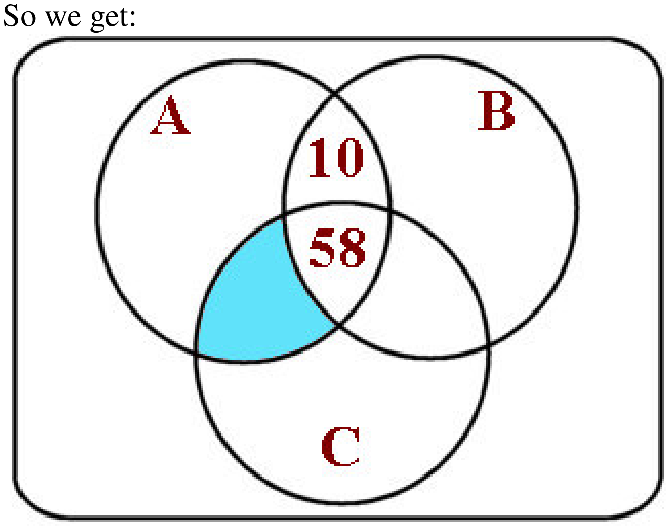
Let A = { students who ate breakfast }
Let B = { students who ate lunch }
Let C = { students who ate dinner }

We begin with just $n(A \cap B \cap C) = 58$:



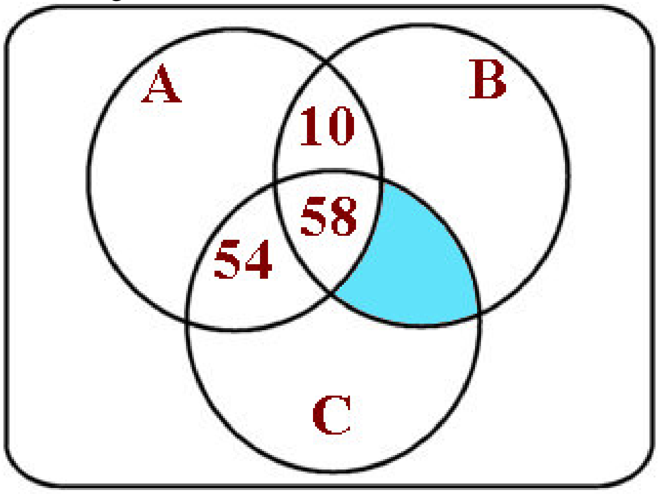
But we know 68 students ate breakfast and lunch so $n(A \cap B) = 58$.
So the above blue region's number must be 68 – 58 = 10.

So we get:



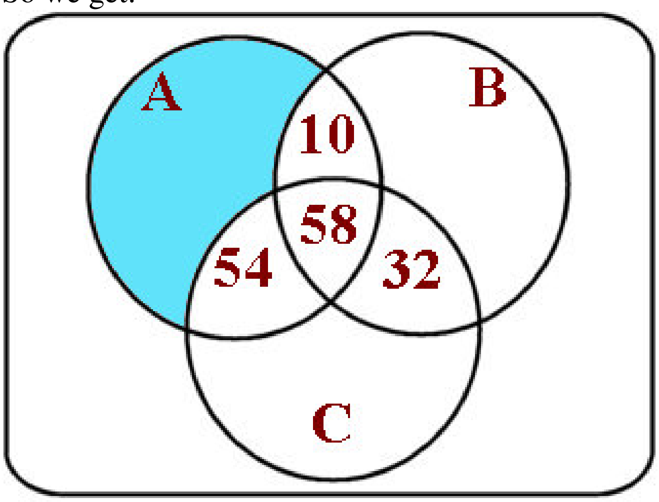
And we know that 112 students ate breakfast and dinner so $n(A \cap C) = 112$.
So the number in the blue region above must be 112 – 58 = 54

And we get:



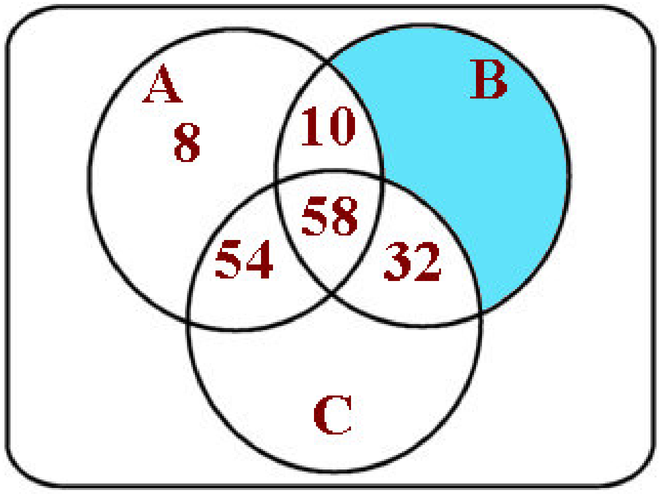
And we know 90 students ate lunch and dinner so n(B ∩ C) = 90.
Thus the number in the blue region above must be 90 – 58 = 32.

So we get:



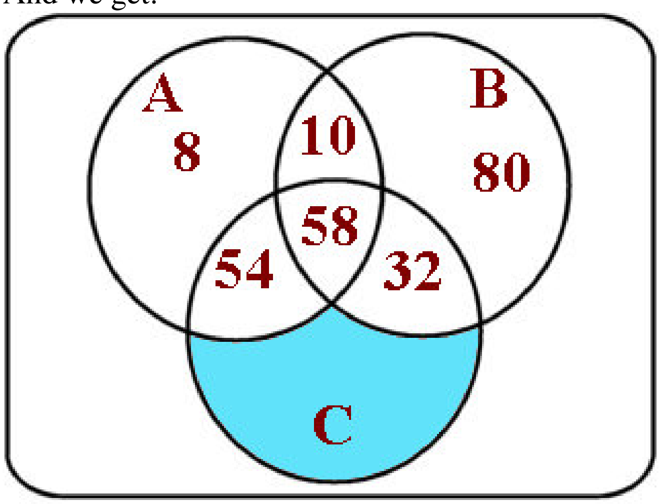
We now note that 130 students ate breakfast. So n(A) = 130.
So the above blue region must be 130 – 10 – 58 – 54 = 8.

Giving us:



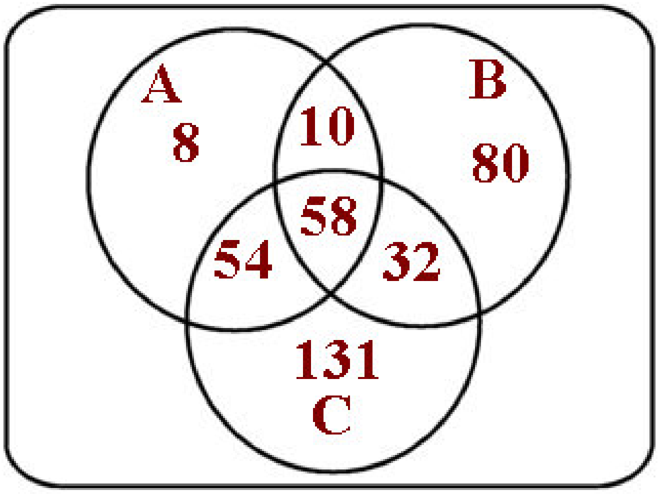
We note that 180 students ate lunch so n(B) = 180
and the above blue region must be 180 – 10 – 58 – 32 = 80

And we get:



Now we recall 275 students at dinner so n(C) = 275
and the above blue region is 275 – 54 – 58 – 32 = 131

And our completed Venn diagram is:

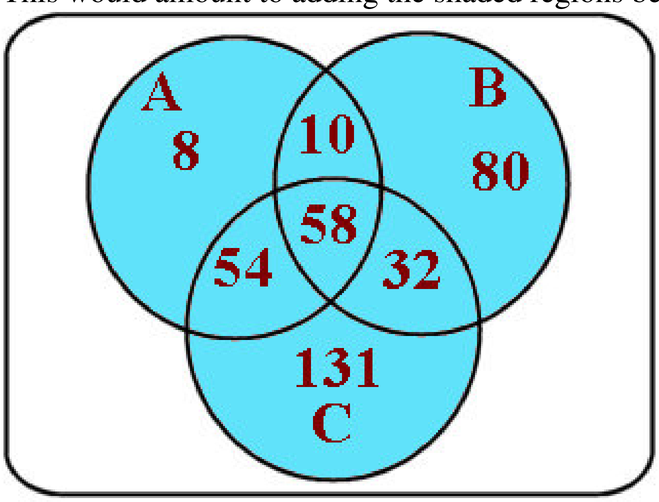


Now we can answer the questions.

34a. How many students ate at least one meal in the cafeteria?

This would be the same as $n(A \cup B \cup C)$

This would amount to adding the shaded regions below:

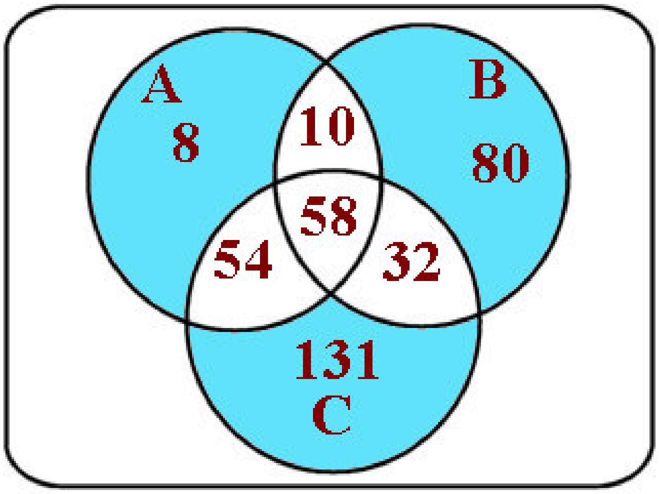


So we get $8 + 10 + 80 + 54 + 58 + 32 + 131 = 373$

So 373 students at least one meal in the cafeteria.

34b. How many students ate exactly one meal in the cafeteria?

This would amount to adding the shaded regions below:

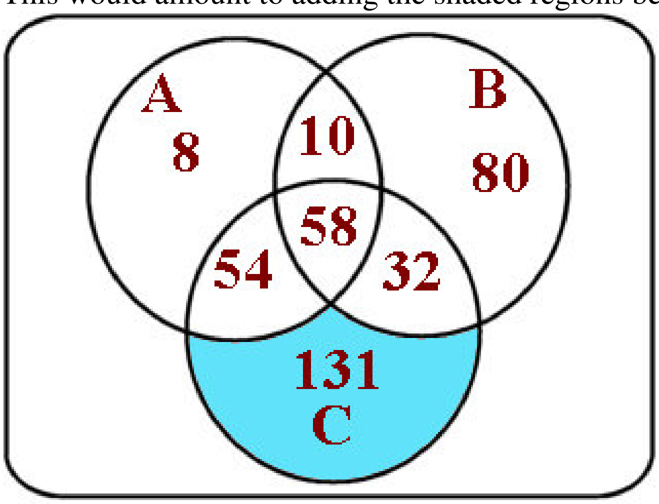


So we get 8 + 80 + 131 = 219

219 students ate exactly one meal in the cafeteria.

34c. How many students ate only dinner in the cafeteria?

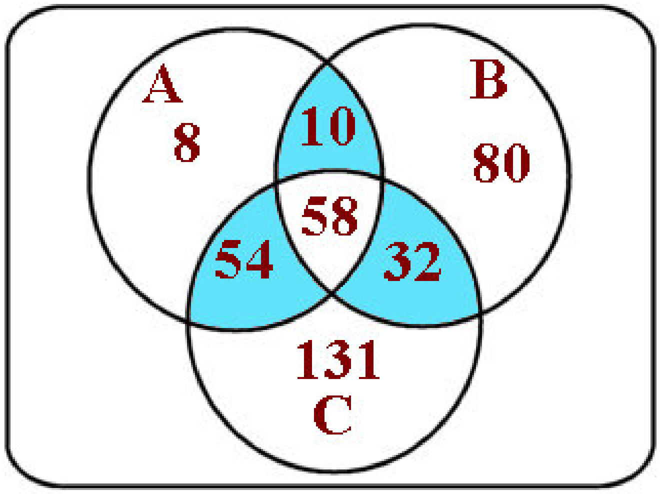
This would amount to adding the shaded regions below:



131 students only ate dinner in the cafeteria.

34d. How many students ate exactly two meals in the cafeteria?

This would amount to adding the shaded regions below:
(Recall: the region with 58 are students who ate all 3 meals)



Which gives us the answer 10 + 54 + 32 = 96

96 students ate exactly two meals in the cafeteria.