Finite Math Section 6_4
Solutions and Hints

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for the book:

Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 6.4:

For this section you should know (at least) 4 formulas:

1. The number of permutations of n distinct objects taken r at a time is:
   $P(n,r) = \dfrac{n!}{(n-r)!}$

2. Permutations of n Objects, not all distinct:
   Given a set of n objects in which k_1 objects are all alike, k_2 objects are all alike, …
   and k_r objects are all alike such that: $k_1 + k_2 + \ldots + k_r = n$,
   then the number of permutations of these n objects taken n at a time is given by:
   $\dfrac{n!}{k_1!k_2!\ldots k_r!}$

3. The number of combinations of n objects taken r at a time is denoted:
   $C(n, r) = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

4. The number of (different) circular arrangements of n distinct objects is $(n - 1)!$

Problem 26:

How many three-digit numbers can be formed using
the numerals in the set { 3, 2, 7, 9 } if repetition is NOT allowed?

So the number to be created has the form: [digit][digit][digit]

For the first digit there are 4 possible choices
For the second there are 3
For the last there are 2

So the answer is
4 * 3 * 2 = **24 three-digit numbers can be formed.**

Problem 39:
Find the number of distinguishable permutations that can be formed from the letters in the word ANTARCTICA.

Notice there are 3 A's, 1 N, 2 T's, 1 R, 2 C's, 1 I.
And a total of 10 letters

So we use the formula for permuting n objects, not all distinct:

$$\frac{10!}{3!*1!*2!*1!*2!*1!} = \frac{10!}{3!*2!*2!} = \frac{3628800}{6*2*2} = \frac{3628800}{24} = 151200$$

So there are
151, 200 distinguishable permutations

Problem 40:
Find the number of distinguishable permutations that can be formed from the letters in the word PHILIPPINES.

Notice there are 3 P's, 1 H, 3 I's, 1 L, 1 N, 1 E, 1 S.
And a total of 11 letters.

So we use the formula for permuting n objects, not all distinct:

$$\frac{11!}{3!*1!*3!*1!*1!*1!*1!} = \frac{11!}{3!*3!} = \frac{39916800}{6*6} = 1108800$$

So there are
1,108,800 distinguishable permutations.

Problem 44:
A group of five students studying for the bar exam has formed a study group. Each member of the group will be responsible for preparing a study outline for one of five courses. In how many different ways can the five courses be assigned to the members of the group?

So basically each member is an empty slot, so there are 5 slots:

___ ___ ___ ___ ___

And each slot needs to be assigned a course.

For the first slot there are 5 options.
For the second there are 4
For the third there 3
For the fourth there are 2
and for the last the remains only 1 course.

So the total number of ways to assign the tests to the members is:
5 * 4 * 3 * 2 * 1 = **120 ways**

Problem 48:
A company car that has a seating capacity of six is to be used by six employees who have formed a car pool. If only four of these employees can drive, how many possible seating arrangements are there for the group?

So there are basically 6 slots, but the first slot (driver's slot) only has 4 possible people. However, the order of seating does matter as we are talking about seating arrangements.

___ ___ ___ ___ ___ ___
Driver

First select the driver -> there are only 4 choices.
Then for the next choice there remain only 5 people to choose from.
The next has only 4 options.
The next has only 3.
The next only 2.
And the last only has 1.

So the total number of seating arrangements is:

4 * 5 * 4 * 3 * 2 * 1 = **480 ways**

Problem 56:

In how many different ways can a panel of 12 jurors and 2 alternate jurors be chosen from a group of 30 prospective jurors?

First we determine how many ways there are to pick 12 jurors – notice the order in which they are picked does NOT matter. So we will choose 12 from 30.

$$C(30, 12) = \frac{30!}{12!(30-12)!} = 86493225$$

Now of the 18 remaining we need to see how many ways there are to choose 2.

$$C(18, 2) = \frac{18!}{2!(18-2!)} = 153$$

So the total number of ways to select 12 jurors and then 2 alternates is:

86,493,225 * 153 = **13,233,463,425 ways**

Think of it this way: There are 86,493,225 paths to the 12 jurors and 153 paths from them to the 2 alternate jurors. So the multiply rule applies.

Problem 58:

A student taking an examination is required to answer 10 out of 15 questions.

a. In how many ways can the 10 questions be selected?

Since order does not matter just calculate 15 choose 10:

$$C(15, 10) = \frac{15!}{10!(15-10)!} = \textbf{3003 ways.}$$

b. In how many ways can the 10 questions be selected if exactly 2 of the first 3 questions must be answered?

Again order does not matter, however we must first select EXACTLY 2 of the first 3 questions. How many ways are there to select 2 from 3? Take 3 choose 2:

$$C(3, 2) = \frac{3!}{2!(3-2)!} = 3$$

And now how many ways are there to select 8 from the remaining 12? (notice we cannot opt to select all 3 of the first 3 – as only exactly 2 were allowed). So we calculate 12 choose 8:

$$C(12, 8) = \frac{12!}{8!(12-8)!} = 495$$

So there are 3 ways to choose the first 2 questions and 495 ways to choose the last 8 questions. Thus the total number of ways to choose the 10 questions is:

3 * 495 = **1485 ways.**

Problem 68:

If a 5-card poker hand is dealt from a well shuffled deck of 52 cards, how many different hands consist of Four of a Kind?

First calculate how many ways you can achieve four of the same rank:
There are 13 different ranks (Ace, 2, 3, …, King). So there are 13 possible ways to get four of a kind.

Now how many ways remain to select the 5^{th} card?
You already have 4 cards from the deck, so only 48 remain.
Thus there are 48 possibilities for the fifth card.

So the total number of ways to get four of a kind is: 13 * 48 = **624 ways**

Problem 70:

If a 5-card poker hand is dealt from a well shuffled deck of 52 cards, how many different hands consist of Two Pairs?

First calculate how many ways you can achieve the first pair:
There are 4 different suits for each rank – you only need two of the same rank.
So there are 4 choose 2 ways to get a pair of any two ranks = C(4, 2) = 6.

There are 13 possible ranks for the first pair in your hand.
So there are 6 * 13 = **78 ways to get the first pair**.

For the second pair there are still 4 choose 2 = 6 ways to get a pair.
However there are only 12 remaining ranks (because you already have 2 of one rank, and if you got the other two of the same rank you would no longer have 2 pairs, but four of a kind) So there 6 * 12 = **72 ways to get the second pair**.

You now have 4 cards in your hand and must get one other card – not of the same rank of either pair in your hand (else you have a full house). So we pretend 8 cards have been removed from the deck – the 4 you have and there associated complements. Thus there remains 52 – 8 = **44 ways to select the last card**.

Thus there are 78 * 72 * 44 = **247,104 ways to get Two Pairs**.

Problem 75:
In how many ways can five TV commentators be seated at a round table for discussion?

You should be aware that the number of (different) circular arrangements of n distinct objects is $(n - 1)!$

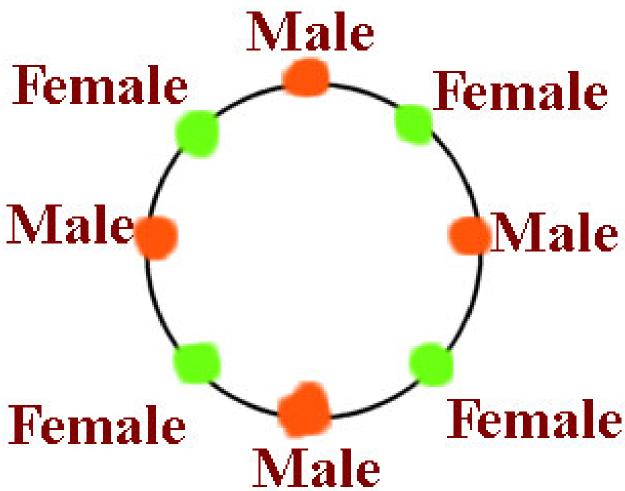
If not, you are now. Simply put 5 into the above equation:

$(n - 1)! = (5 - 1)! = 4! =$ **24 ways**.

Problem 76:
In how many ways can four men and four women be seated at a round table at a dinner party if each guest is seated next to members of the opposite sex?

You should be aware that the number of (different) circular arrangements of n distinct objects is $(n - 1)!$

Now consider the picture:



So you should see that we need to break this into finding how many ways we can seat 4 men in a circular arrangement. And then determine how many ways there are to seat 4 women in a circular arrangement (which is likely to be the same number of ways as 4 men).

First we see there (4 – 1)! = 6 ways to seat four men in a circle.
Likewise there are 6 ways to seat four women in a circle.

Thus we discover there are
6*6 = **36 ways to seat the eight people**.