Finite Math Section 7_2 Solutions and Hints

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for the book: <u>Finite Mathematics, 7th Edition</u> by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN **!!!!!!!** This is designed to assist you in the event you get stuck. If you do not do the work you will NOT pass the tests.

Section 7.2:

For this section you should know some definitions and stuff:

- 1. The **probability of an event ALWAYS** is a number between 0 and 1. There are no exceptions to this it is part of the definition.
- 2. Know what a **probability distribution** is (usually expressed as a table be able to read and create such tables).
- 3. Know what a **probability function** is (function which given an event returns the probability of the event occurring).
- 4. A <u>Uniform Sample Space</u> means that each event (outcome) has the same chance of occurring as any other event (outcome).

This section can be confusing. The book says many confusing things. The key point is given an experiment there always is a total number of things that can happen.

Assuming a Uniform Sample Space (i.e. each thing has the same chance of occurring as any other) then the probability of a thing occurring is simply the number of ways it might happen divided by the total number of things that can happen.

Sometimes a particular event is more likely to occur than another. For example if you painted 5 sides of a die red and the remaining black you would expect to "roll red" more often than "rolling black." Specifically you would expect to "roll red" five out of every six times (on average) and to "roll black" 1 out of every 6 times. To figure this out you note that there are 6 total outcomes. Five of the outcomes are red and only 1 is black.

Problem 12:

The arrival times of the 8 A.M. Boston-based commuter train as observed in the suburban town of Sharon over 120 weekdays is summarized below:

Arrival time = x	Frequency of Occurrence
7:56 a.m. < x ≤ 7:58 a.m.	4
7:58 a.m. < x ≤ 8:00 a.m.	18
8:00 a.m. < x ≤ 8:02 a.m.	50
8:02 a.m. < x ≤ 8:04 a.m.	32
8:04 a.m. < x ≤ 8:06 a.m.	9
8:06 a.m. < x ≤ 8:08 a.m.	4
8:08 a.m. < x ≤ 8:10 a.m.	3

12a. Describe an appropriate sample space for this experiment.

This is just a set containing 7 elements of each time interval.

12b. Find the empirical probability distribution for this experiment.

Don't panic about the word empirical – it just means from the experimental data as recorded in the table. Empirical evidence is evidence observed.

All you do for this is make another table: Where the probability of an event occurring = (it's frequency) / (total events). Note that the total number of events (observations) = 120. (add all the numbers up to check it)

Simple Event,	probability of Arrival time = x,
Arrival time = x	p(Arrival time = x)
$7:56 \text{ a.m.} < x \le 7:58 \text{ a.m.}$	$p(7:56 a.m. < x \le 7:58 a.m.) = 4 / 120$
7:58 a.m. $< x \le 8:00$ a.m.	$p(7:58 a.m. < x \le 8:00 a.m.) = 18 / 120$
$8:00 \text{ a.m.} < x \le 8:02 \text{ a.m.}$	$p(8:00 \text{ a.m.} < x \le 8:02 \text{ a.m.}) = 50 / 120$
$8:02 \text{ a.m.} < x \le 8:04 \text{ a.m.}$	$p(8:02 \text{ a.m.} < x \le 8:04 \text{ a.m.}) = 32 / 120$
$8:04 \text{ a.m.} < x \le 8:06 \text{ a.m.}$	$p(8:04 a.m. < x \le 8:06 a.m.) = 9 / 120$
$8:06 \text{ a.m.} < x \le 8:08 \text{ a.m.}$	$p(8:06 a.m. < x \le 8:08 a.m.) = 4 / 120$
$8:08 \text{ a.m.} < x \le 8:10 \text{ a.m.}$	$p(8:08 a.m. < x \le 8:10 a.m.) = 3 / 120$

Some instructors will want you to express the above fractions as decimals. Be sure to ask your instructor which way to write the answer.

Problem 22:

One light bulb is selected at random from a lot of 120 light bulbs, of which 5% are defective. What is the probability that the selected light bulb is defective?

So you have 120 light bulbs total. 120 * 0.05 = 6 are defective.

So there is a 6 in 120 chance of selecting a defective bulb. Or rather,

The probability of selecting a defective bulb is 6 / 120.

Problem 24:

If a ball is selected at random from an urn containing three red balls, two white balls and five blue balls, what is the probability that it will be a white ball.

So you have 3 + 2 + 5 = 10 balls total. Two of those 10 are blue. So you have a 2 in 10 chance of selecting a white ball. Or rather,

The probability of selecting a white ball is 2 / 10.

Problem 26:

A pair of fair dice is cast. What is the probability that:

26a. The sum of the numbers shown uppermost is less than 5?

For this make a table of the possible outcomes of number pairs and note which pairs sum to less than 5 (they are highlighted in green):

1	1	1 2	1 3	1 4	1 5	1 6
2	1	2 2	2 3	2 4	2 5	2 6
3	1	3 2	3 3	3 4	3 5	3 6
4	1	4 2	4 3	4 4	4 5	4 6
5	1	5 2	5 3	5 4	5 5	56
6	1	6 2	6 3	6 4	6 5	6 6

There are 6 ways that the dice could add up to less than 5. Notice there are 36 total outcomes possible.

So the answer is 6/36.

26b. At least one 6 is cast?

Again make a table of all the possible outcomes and count the number of ways that at least one of the two numbers is a 6:

1 1	1 2	1 3	1 4	1 5	16
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	56
6 1	6 2	6 3	6 4	6 5	6 6

So there are 11 possible outcomes where at least one of the dice shows a 6. There are a total of 36 possible outcomes.

So the probability that at least one 6 is cast is 11/36.

Problem 42:

The number of planes in the fleets of five leading airlines that contain Airfones is shown in the below table:

Airline	Planes with Airfones	Size of Fleet
Α	50	295
В	40	325
С	31	167
D	29	50
E	25	248

42a. If a plane is selected at random from airline A, what is the probability that it contains an Airfone?

Airline A has 295 total planes. Fifty of the planes have Airfones. So 50 of every 295 planes will have an airfone. Or rather,

The probability of a plane of Airline A having an Airfone is 50 / 295.

42b. If a plane is selected at random from the entire fleet of the 5 airlines, what is the probability that it contains an Airfone?

There is a total of 295+325+167+50+248 = 1085 planes. Of those 50+40+31+29+25 = 175 have Airfones.

The probability of a plane selected from all of them having an Airfone is 175 / 1085.