Finite Math Section 7_3
Solutions and Hints

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for the book:
<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 7.3:

For this section notice E and F almost always represents events (sets containing possible outcomes of an experiment) and S is the entire sample space of an experiment (a list of all possible outcomes).

There are 5 basic properties presented in this section. I have reduced it to 4 because properties 3 and 4 in the book are the basically the same thing. Some instructors use these properties directly, others do not. If you plan on taking any more courses involving probability you should memorize these properties.

1. $P(E) \geq 0$ for any E
2. $P(S) = 1$ where S = the entire sample space
3. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
   Recall if E and F are mutually exclusive $P(E \cap F) = 0$.
4. $P(E^C) = 1 - P(E)$

Do as many of these type of problems as possible. Sometimes they are tricky and sometimes they are not. You will need to be able to tell how to solve a variety of them. The only way to learn is to do.

<u>Problem 6</u>:
A pair of dice is cast and the number that appears uppermost on each die is observed. Find the probability that the sum of the numbers is at least 4.

This problem is a review of section 6.2. Work it exactly as you did there using a table. Select each pair that sums to at least 4.

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

So 33 pairs of a total of 36 sum to at least 4. So the probability of the dice summing to at least 4 is **33 / 36**.

You may also note that each pair in the table has a probability of 1/36 of occurring and that you can sum the probabilities of the "successful" pairs:
1/36 + 1/36 = **33 / 36**

<u>Problem 26</u>:
Let E and F be two events of an experiment with sample space S.
Suppose $P(E) = 0.6$, $P(F) = 0.4$ and $P(E \cap F) = 0.2$.

26a. Compute $P(E \cup F)$
Use the formula $P(E \cup F) = P(E) + P(F) – P(E \cap F)$. So:
$P(E \cup F) = 0.6 + 0.4 – 0.2$ → $\mathbf{P(E \cup F) = 0.8}$

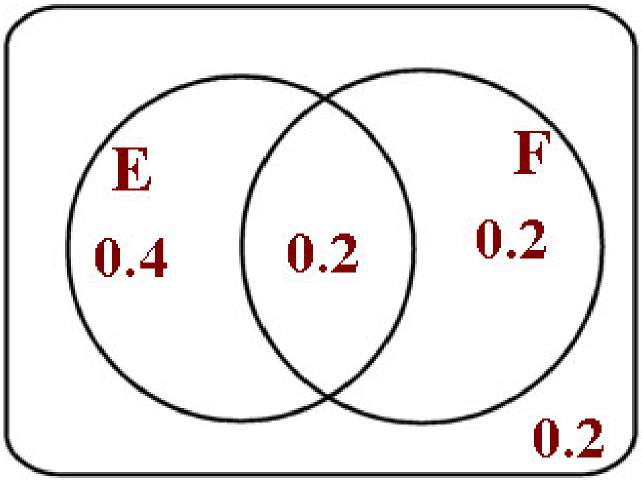
26b. Compute $P(E^C)$
Use the formula $P(E^C) = 1 – P(E)$, so:
$P(E^C) = 1 – 0.6$ → $\mathbf{P(E^C) = 0.4}$

26c. Compute $P(F^C)$
Use the formula $P(F^C) = 1 – P(F)$, so:
$P(F^C) = 1 – 0.4$ → $\mathbf{P(F^C) = 0.6}$

26d. Compute $P(E^C \cap F)$

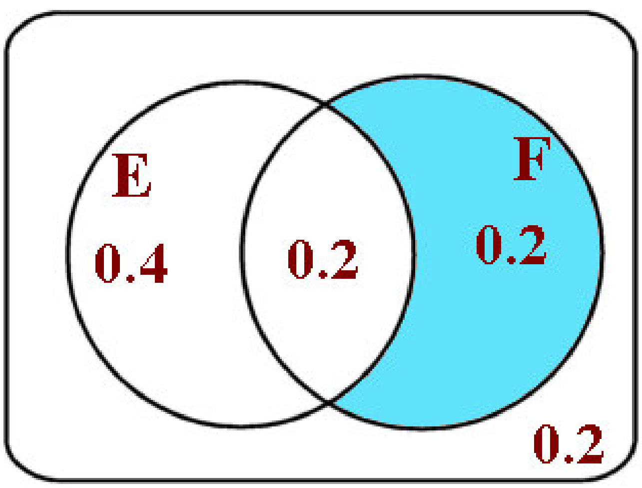
You should want to use the formula $P(E^C \cup F) = P(E^C) + P(F) - P(E^C \cap F)$.
But what is $P(E^C \cap F)$?

To figure that out you need to create a diagram to help you.
Use the facts that $P(E) = 0.6$, $P(F) = 0.4$ and $P(E \cap F) = 0.2$. Recall that $P(S) = 1$.



Notice that there is a 0.2 chance that neither E nor F occurs. (This is found by seeing that $0.4 + 0.2 + 0.2 = 0.8$ and $1 - 0.8 = 0.2$)

So what is the $P(E^C \cap F)$? Shade the diagram representing $E^C \cap F$ and you will get:



So $P(E^C \cap F) = 0.2$ and using the formula $P(E^C \cup F) = P(E^C) + P(F) - P(E^C \cap F)$ gives:
$P(E^C \cup F) = 0.4 + 0.4 - 0.2$

$P(E^C \cup F) = 0.6$

Can you see an easier way to calculate this straight from the Venn Diagram?

Problem 30:
Among 500 freshmen pursuing a business degree at a university, 320 are enrolled in an Economics course, 225 are enrolled in a Math course and 140 are enrolled in both an Economics and Math course. What is the probability that a randomly selected freshman from this group is enrolled in:

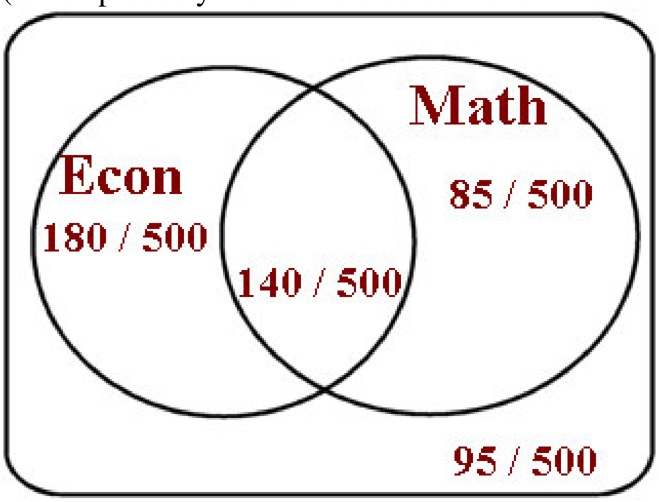
First let's set some stuff up:

Let E = the event that the freshman is enrolled in an Economics course.
Let M = the event that the freshman is enrolled in a Math course.
So
$P(E) = 320 / 500$
$P(M) = 225 / 500$
$P(E \cap M) = 140 / 500$

Notice to get "only enrolled in Econ" we subtract 140 from 320 = 180.
Likewise to get "only enrolled in Math" we subtract 140 from 225 = 85.

Notice also that
(total students) – (just in Econ) – (in both Econ and Math) – (just in math) gives:
500 – 180 – 140 – 85 = 95 students are neither enrolled in Econ nor Math.

From this we derive the below Venn Diagram
(which probably makes more sense then all the above words):



30a. An Economics and/or a Math course?
This is just
$P(E \cup M) = P(E) + P(M) - P(E \cap M)$
$= 320/500 + 225/500 - 140/500 =$ **405 / 500**

Or you an use the above Venn Diagram and get
180/500 + 140/500 + 85/500 = **405 / 500**

30b. Exactly one of these two course?
This I would just look at the above diagram to get:
180/500 + 85/500 = **265 / 500**

30c. Neither an Economics nor a Math course?
Again just referring to the above diagram I would get:
95 / 500

Notice that "neither an Econ nor a Math" is the same thing a the complement of the union of Econ and Math:
$P((E \cup M)^C) = 1 - P(E \cup M) = 1 - 495 / 500 = 95 / 500$.

<u>Problem 34</u>:
In a survey conducted to see how long Americans keep their cars, 2000 automobile owners were asked how long they plan to keep their present cars. The results of the survey follow:

Years Car Is kept = x	Respondents
$0 \le x < 1$	60
$1 \le x < 3$	440
$3 \le x < 5$	360
$5 \le x < 7$	340
$7 \le x < 9$	240
$x \ge 10$	560

Find the probability distribution associated with this data. What is the probability that an automobile owner selected at random from those surveyed plans to keep his or her present car:

34a. Less than five years?
First we need to set up the probability distribution table
Note there were 2000 people surveyed and the sum of the Respondents column = 2000:

Years Car Is kept = x	Respondents	P(x)
$0 \le x < 1$	60	60 / 2000
$1 \le x < 3$	440	440 / 2000
$3 \le x < 5$	360	360 / 2000
$5 \le x < 7$	340	340 / 2000
$7 \le x < 10$	240	240 / 2000
$x \ge 10$	560	560 / 2000

So the probability of the person saying they will keep it less than five years is
(the probability they would keep it for less than a year)
+ (the probability they would keep it for 1 to almost 3 years)
+ (the probability they would keep it for 3 to almost 5 years)

Or rather:

$p(0 \le x < 1) + p(1 \le x < 3) + p(3 \le x < 5) \quad = 60/2000 + 440/2000 + 360/2000$

$= \mathbf{860 / 2000}$

34b. Three or more years?

This is just a sum of the probabilities of keeping it (from 3 to almost 5 years) + (5 to almost 7 years) + (7 to almost 10 years) + (10 or more years). Or rather:

$p(3 \le x < 5) + p(5 \le x < 7) + p(7 \le x < 10) + p(x \ge 10)$

$= 360/2000 + 340/2000 + 240/2000 + 560/2000$

$= \mathbf{1500 / 2000}$