Finite Math Section 7_4
Solutions and Hints

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for the book:
<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 7.4:

For this section we incorporate "counting" with probability. The important thing to remember here is that if order does NOT matter when selecting your objects then use a combination (i.e. C(n, r)) to determine the number of ways to select the objects.

For example if you are going to pick 2 balls out of 10 then there are C(10, 2) = 45 ways to do so. Because the ORDER the balls are selected in does NOT matter.
Do NOT simply say there are 10*9 ways to select the 2 balls as order does not matter this is incorrect.

I know this is confusing. A better example is selecting teams. Say you have 10 people who want to play along with yourself and another team captain. The other team captain generously allows you to select your entire team and will take whoever is left over. So you get to pick 5 people out of 10. The order you select Tom, Bob, Harry, John and Mike does NOT matter. What matters is that all 5 of them end up on your team. So the question really is how many possible teams of 5 could you create.

Hopefully that helps. The basic idea for figuring out probabilities is still the same:
First figure out how many total ways there to do something = n(S)
Then figure out how many ways there are to do something specific = n(E)
Finally you say the probability = n(E) / n(S).

Do many of these problems. They will appear on tests. They require experience.

<u>Problem 1</u>:
An unbiased coin is tossed 5 times.
Find the probability that it lands heads all five times.

So the total number of outcomes = 2*2*2*2*2 = 32 = P(S).
(For each toss there are 2 outcomes)

There is only 1 way it can land heads all 5 times. So 1 = P(E).
(Notice C(5, 5) =1)

Thus the probability is **1 / 32**.

<u>Problem 2</u>:
An unbiased coin is tossed 5 times.
Find the probability that it lands heads exactly once.

So the total number of outcomes = 2*2*2*2*2 = 32 = P(S).

There are 5 ways it could land heads exactly once, so P(E) =5.
(Notice C(5, 1) = 5)

Thus the probability is **5 / 32**.

<u>Problem 4</u>:
An unbiased coin is tossed 5 times.
Find the probability that it lands heads more than once.

Notice this is just the opposite (complement) of problem 2's question.
Or rather.
If
E = {coins lands heads exactly once}
Then
E^C = { coin doesn't land heads exactly once } = { coin lands heads more than once }

Recall the total probability of all possible events = 1, so:
$P(E) + P(E^C) = 1$
p(exactly once) + p(more than once) = 1
5 / 32 + p(more than once) = 1
p(more than once) = 1 – 5/32
p(more than once) = **27 / 32**

Problem 8:

Two cards are selected at random without replacement from a well shuffled deck of 52 playing cards. Find the probability that two cards of the same suit are drawn.

First realize that there are 13 cards of each suit
(Ace, 2, 3, 4, 5, 6, 7, 8, 9, Jack, Queen, King).

Second realize there are 4 suits: Clubs, Spades, Hearts and Diamonds.

Now let us solve an easier problem: What is the probability the two cards are both clubs?

Let n(E) = number of ways both cards are clubs
= number of ways to select 2 cards from 13 (order does NOT matter)
= C(13, 2)
= 78

Let n(S) = total number of ways to select 2 cards from deck of 52.
= C(52, 2)
= 1326

So the probability the two cards are both clubs = 78 / 1326.

Now we will solve another easy problem:
What is the probability the two cards are both spades?

Let n(E) = number of ways both cards are spades
= number of ways to select 2 cards from 13 (order does NOT matter)
= C(13, 2)
= 78

Let n(S) = total number of ways to select 2 cards from deck of 52.
= C(52, 2)
= 1326

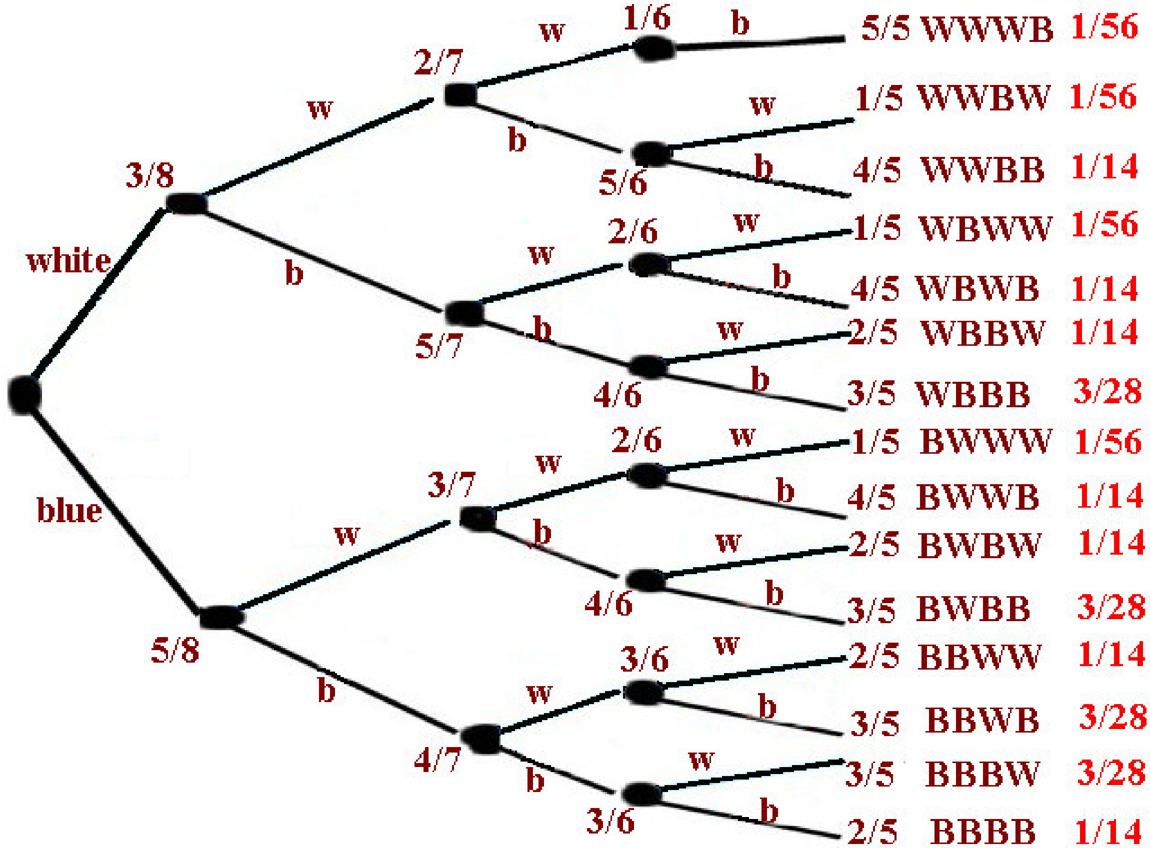
So the probability the two cards are both spades = 78 / 1326.

Likewise we will find the probability both cards are hearts =78 / 1326 and the probability that both cards are diamonds = 78 / 1326.

From this we can find the answer to the actual question of what is the probability that both cards are of the same suit by realizing:
p(both same suit) = p(both clubs) + p(both spades) + p(both hearts) + p(both diamonds)
p(both same suit) = 78 / 1326 + 78 / 1326 + 78 / 1326 + 78 / 1326
p(both same suit) = **312 / 1326** = 4 / 17

Problem 9:
Four balls are selected at random without replacement from an urn containing 3 white balls and 5 blue balls.
Find the probability two of the balls are white and two are blue.

To solve this you can use the combination ideas, but I am first going to present a different method, which is sometimes easier to understand: First we are going to make a probability tree, which lists all possible events (outcomes):



Notice there are 16 possible outcomes. Each outcome has its probability listed to the right of it. These are calculated by multiplying the numbers along the path to the outcome. For example consider the topmost path with outcome WWWB (white, white, white, blue). The probability for this outcome is 1/56. This can be seen by the following reasoning: The odds of first choosing a white ball are 3/8. The odds of choosing a white ball for the second choice are then 2/7 (as you already picked 1 white ball only 2 remain). The odds of picking a white ball as the third choice are 1/6. The five remaining balls are all blue so the odds of picking a blue ball as the fourth choice are 5/5. Using the multiplication rule we see the probability of the event WWWB is: 3/8*2/7*1/6*5/5 = 1/56.

The question asked is what is the probability 2 of the balls are blue and two are white. From the tree we count such events: WWBB, WBWB, WBBW, BWWB, BWBW, BBWW so we see there are 6 ways, each with probability 1/14. So

p(2 balls white and 2 blue) = 1/14 + 1/14 + 1/14 + 1/14 + 1/14 + 1/14 = 6/14 = **3/7**.

The alternate way to solve this problem is to say:
How many ways are there to choose 2 white balls from 3 = C(3, 2) = 3
How many ways are there to choose 2 blue balls from 5 = C(5, 2) = 10
How many ways are there to choose 4 balls from 8 = C(8, 4) = 70.

So the solution is (num ways to pick 2 white) * (ways to pick 2 blue) / (total ways)
= 3*10 / 7 = **3/7.**

Problem 10:
Four balls are selected at random without replacement from an urn containing 3 white balls and 5 blue balls.
Find the probability all the balls are blue.

From the tree (in problem 9) we see there is only one way for this to happen with probability of 5/8 * 4/7 * 3/6 * 2/5 = **1 / 14**.

Our you can use the following questions:
How many ways are there to choose 4 blue balls from 5 = C(5, 4) = 5.
How many ways are there to choose 4 balls from 8 = C(8, 4) = 70.

And the solution is 5 / 70 = **1 / 14**.

Problem 11:
Four balls are selected at random without replacement from an urn containing 3 white balls and 5 blue balls.
Find the probability exactly three of the balls are blue.

From the tree we see there are 4 ways this outcome could occur (WBBB, BWBB, BBWB, BBBW) each with a probability of 3/28. So the answer is the sum:
p(WBBB) + p(BWBB) + p(BBWB) + p(BBBW)
= 3/28 + 3/28 + 3/28 + 3/28 = **3 / 7**.

Or you can solve the problem with the questions:
How many ways are there to choose 1 white balls from 3 = C(3, 1) = 3
How many ways are there to choose 3 blue balls from 5 = C(5, 3) = 10
How many ways are there to choose 4 balls from 8 = C(8, 4) = 70.

So the solution is
(num ways to pick 1 white) * (ways to pick 3 blue) / (total ways) = 3*10 / 7 = **3 / 7.**

Problem 12:

Four balls are selected at random without replacement from an urn containing 3 white balls and 5 blue balls.
Find the probability two or three of the balls are white.

From the tree we see there are 4 ways we could get 3 balls being white (WWWB, WWBW, WBWW, BWWW) each with a probability of 1/56. So
p(3 balls white) = 1/56 + 1/56 + 1/56 + 1/56 = 1/14

From problem 9 we know the
p(2 balls white) = 3/7

So p(2 or 3 balls white) = 1/14 + 3/7 = **1 / 2**.

Problem 22:

Electronic baseball games manufactured by Tempco Electronics are shipped in lots of 24. Before shipping, a quality control inspector randomly selects a sample of 8 from each lot for testing. If the sample contains any defective games the entire lot is rejected. What is the probability that a lot containing exactly 2 defective games will be shipped.

Rephrasing this question slightly we will say: what is the probability that none of the eight randomly selected toys is defective?

First we see the total number of ways to select 8 toys from 24 = C(24, 8) = 735,471
Then we see that if we do NOT select either of the two defective toys then we have only 22 toys to choose from. The number of ways to choose 8 from 22 is C(22, 8) = 319,770

So the probability that neither defective toy is detected is 319770 / 753471 = **10 / 23**.

Problem 39:

If a 5-card poker hand is dealt from a well shuffled deck of 52 cards, what is the probability of being dealt a full house?

Note a full house is three of a kind and two of a kind.

First notice there are 13 possible values to have 3 of a kind of (A, 2, 3… King)
Notice there are C(4, 3) = 4 ways to get 3 aces.
Likewise there are 4 ways to get 3 twos, 4 ways to get 3 threes…
So there are a total of 13*4 = **52** ways to get 3 of a kind.

Now notice there are 12 remaining possible values to have 2 of a kind.
Note there are C(4, 2) = 6 ways to get 2 aces.
Likewise there are 6 ways to get 2 twos, 6 ways to get 2 threes…
So there are a total of 12 * 6 = **72** ways to get 2 of a kind.

Lastly notice there are C(52, 5) = **2,598,960** different hands that could be dealt.

So the probability of getting a full house is:
52*72 / 2598960 = 3744/2598960 = **6 / 4165**

Problem 40:
If a 5-card poker hand is dealt from a well shuffled deck of 52 cards, what is the probability of being dealt two pair?

First notice there are 13 possible values to have the first pair of (A, 2, 3… King)
Note there are C(4, 2) = 6 ways to get 2 aces.
Likewise there are 6 ways to get 2 twos, 6 ways to get 2 threes…
So there are a total of 13 * 6 = **78** ways to get the first pair.

Now notice there are 12 remaining possible values to have the second pair be.
Note there are C(4, 2) = 6 ways to get 2 aces.
Likewise there are 6 ways to get 2 twos, 6 ways to get 2 threes…
So there are a total of 12 * 6 = **72** ways to get the second pair.

And notice that leaves 52 – 8 = **46** ways to select the last card.
(we subtract the 4 cards already selected along with their matching 4 counterparts so we don't accidentally get a full house)

Lastly notice there are C(52, 5) = **2,598,960** different hands that could be dealt.

So the probability of getting a full house is:
78*72*46 / 2598960 = 258336 / 2598960 = **414 / 4165**