Finite Math Section 7_5
Solutions and Hints

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for the book:
<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 7.5:

This section speaks on Conditional Probability, so know what that means. =)

Definition:
If A and B are events in an experiment and $P(A) \neq 0$, then $P(B|A) = P(A \cap B) / P(A)$.
From this you can just multiply P(A) to the other side and get $P(A \cap B) = P(B|A)*P(A)$
P(B|A) is read: "probability of B happening given that A is known to have happened."

You must also know:
Two events A and B are independent if and only if $P(A \cap B) = P(A)*P(B)$.
Think about it if the chances of A happening and B happening are the same as if A happened and then B happened obviously A happening has no effect on whether B happens and B has no effect on whether A happens. This can be extended to as many events as you like. EX: A B and C are independent iff $P(A \cap B \cap C) = P(A)*P(B)*P(C)$ and so on.

This section also shows you how to draw cool tree diagrams useful in computing the conditional probabilities. These diagrams have proven to be much easier to understand than working everything out in your head. I would recommend using them (though some problems may not need them, you may get partial credit if you try to draw one =)

You may also need to recall from 7.3:

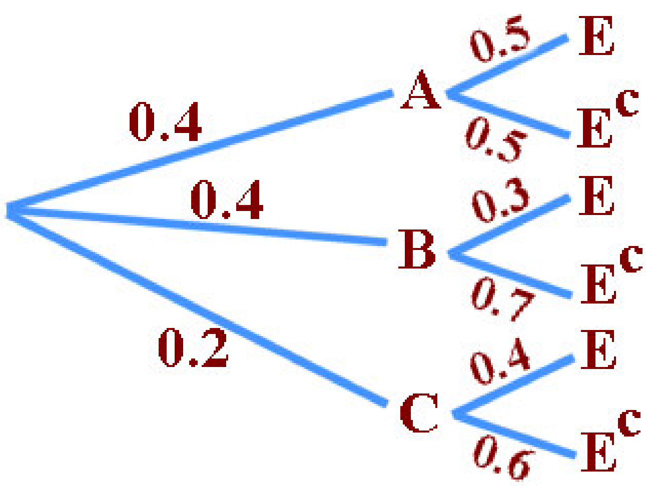
1. **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$**
2. **$P(A^C) = 1 - P(A)$**

<u>Problem 10</u>:
If A and B are independent events, P(A) = 0.35 and P(B) = 0.45 find:

10a. P(A ∩ B)
We are given the events are independent thus
$P(A \cap B) = P(A) * P(B) = 0.35 * 0.45 = \mathbf{0.1575}$

10b. P(A ∪ B)
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.35 + 0.45 - 0.1575 = \mathbf{0.6425}$

<u>Problem 12</u>:
The below tree diagram represents an experiment consisting of two trials. Use the diagram to answer the questions (a) – (f).



12a. P(A)
From the line that leads to A we see that $\mathbf{P(A) = 0.4}$.

12b. P(E|A)
So we assume A happened. So we start at A. From A we see that there is a 0.5 chance that E will happen. So $\mathbf{P(E|A) = 0.5}$.

12c. P(A ∩ E)
So both A and E must have happened. So we trace the path that cause A and E to happen (follow along the top) and we see the path values are 0.4 and 0.5, so
$P(A \cap E) = 0.4 * 0.5 = \mathbf{0.2}$.

12d. P(E)

For this we see there are three ways (paths) that E might occur (one through A, one through B and one through C).

The path through A has probability = 0.4 * 0.5 = 0.2

The path through B has the probability of 0.4 * 0.3 = 0.12

The path through C has the probability of 0.2 * 0.4 = 0.08

P(E) = 0.2 + 0.12 + 0.08 = **0.4**

12e. Does P(A ∩ E) = P(A)*P(E) ?

From part (c) P(A ∩ E) = 0.2.

From part (a) P(A) = 0.4

From part (d) P(E) = 0.4

Clearly 0.2 ≠ 0.4*0.4 so **No**

12f. Are A and E independent events?

Since P(A ∩ E) ≠ P(A)*P(E) the answer is **No**.

Problem 17:

A pair of fair dice is cast. What is the probability that the sum of numbers falling uppermost is less than 9, if it is known that one of the numbers is a 6?

Let A denote the event that the sum is less than nine.

Let B denote the event that one of the numbers is a 6.

A = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5),
(4, 1), (4, 2), (4, 3), (4, 4),
(5, 1), (5, 2), (5, 3),
(6, 1), (6, 2) }

B = { (1, 6), (2, 6), (3, 6), (4, 6), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

so

A ∩ B = { (1, 6), (2, 6), (6, 1), (6, 2) }

thus

p(A ∩ B) = 4 / 36 and p(B) = 11 / 36.

and we know

p(A|B) = p(A ∩ B) / p(B) = (4/36) / (11/36) = **4 / 11**.

<u>Problem 18</u>:
A pair of fair dice is cast. What is the probability that the number landing uppermost on the first die is a 4, if it is known that the sum of the numbers falling uppermost is 7?

Let A denote the event that the first die is a 4.
Let B denote the event that the sum of the dice is 7.

$A = \{ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \}$

$B = \{ (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) \}$

so

$A \cap B = \{ (4, 3) \}$

and we see

$p(A \cap B) = 1 / 36$ and $p(B) = 6 / 36$

And

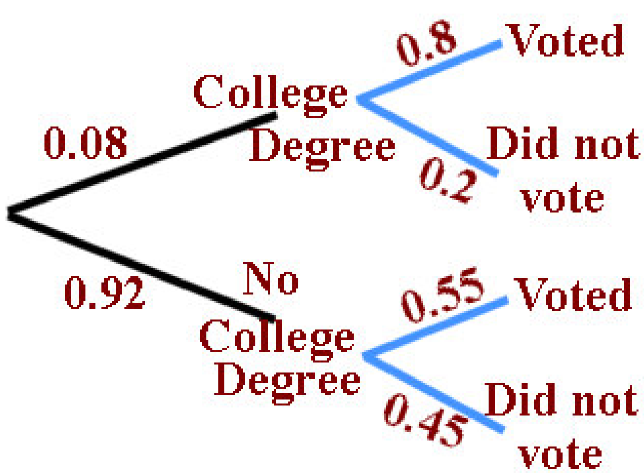
$p(A|B) = p(A \cap B) / p(B) = (1/36) / (6/36)$

$p(A|B) = \mathbf{1 / 6}$.

This should make since: There are 6 ways the dice could add up to seven, only in one of those ways is the first die showing a 4.

<u>Problem 28</u>:
In a survey of 1000 eligible voters selected at random, it was found that 80 had a college degree. Additionally it was found that 80% of those who had a college degree voted in the last presidential election, whereas 55% of the people who did not have a college degree voted in the last presidential election. Assuming that the poll is representative of all eligible voters find the probability that an eligible voter selected at random:

28a. Had a college degree and voted in the last presidential election.
For these questions first make a tree: Note that 80 of 1000 is 0.08.



So $P(\text{college degree} \cap \text{voted}) = 0.08 * 0.8 = \mathbf{0.064}$

28b. Did not have a college degree and did not vote in the last presidential election.

From the tree we easily see:

P(no degree $\cap$ no vote) = 0.92 * 0.45 = **0.414**

28c. Voted in the last presidential election.

Notice that P(voted) = P(degree AND voted) + P(no degree AND voted).
So from the tree we get:

P(degree AND voted) = 0.08*0.8 = <u>0.064</u>
P(no degree AND voted) = 0.92*0.55 = <u>0.506</u>

Thus

P(voted) = 0.064 + 0.506 = **0.57**

28d. Did not vote in the last presidential election.

You could work this out the long way

P(no vote) = P(degree AND no vote) + P(no degree AND no vote)
= 0.08 * 0.2 + 0.92*0.45
= 0.016 + 0.414
= 0.43

Or you could just say "Hey this is the complement of question c" and find the answer by saying

P(no vote) = 1 – P(vote) = 1 – 0.57 = **0.43**