Finite Math Section 7_6
Solutions and Hints

by Brent M. Dingle

for the book:
<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

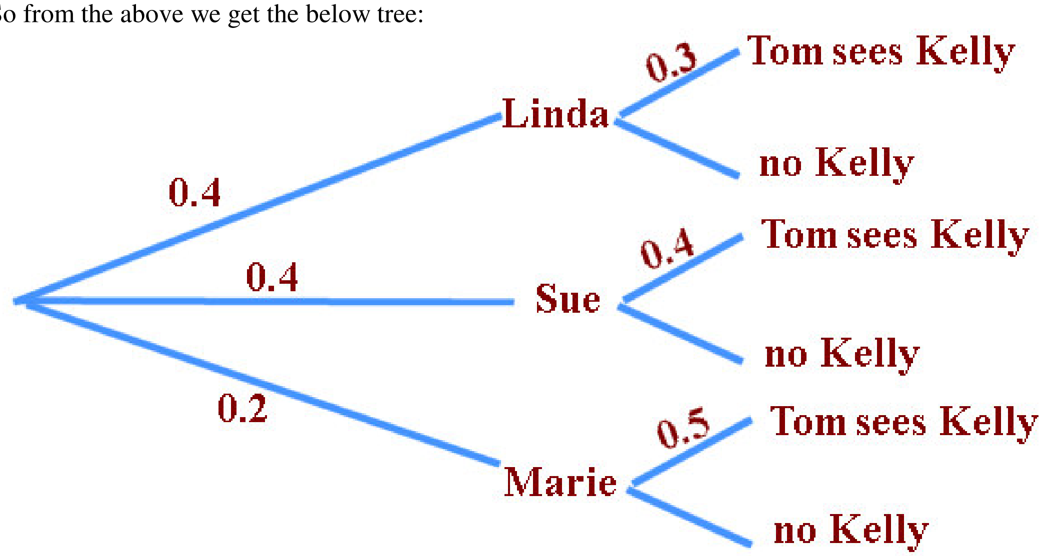
DO NOT PRINT THIS OUT AND TURN IT IN ! This is designed to assist you in the event you get stuck. If you do not do the work you will NOT pass the tests.

Section 7.6:

This section is all about Bayes Theorem. Written out it looks really complicated. Don't be scared by that. Look at the tree diagrams – they are created as they have been in the last couple sections. Now just watch what is done with the numbers.

In simple words all you do is pick the path you are curious about and calculate its probability. Now look at all the other paths that end in the same event as the one you are interested in. Sum the probabilities of all these other paths AND the one of interest. Divide the probability of your chosen path by the sum and "poof" you have your answer.

For example consider that Bob has 3 choices go out with Linda, Sue or Marie. Because Bob likes Linda and Sue about the same and Marie a little less the chance of Bob going out with Linda is 0.4 and the chance of him going out with Sue is 0.4 but the chance of him going out with Marie is on 0.2. Now if Bob goes out with Linda then there is a 0.3 chance that Tom will go out with Kelly. If Bob goes out with Sue there is 0.4 chance that Tom will go out with Kelly. And if Bob goes out with Marie then there is a 0.5 chance that Tom will go out with Kelly. (Don't ask why Tom going out with Kelly is even related to who Bob goes out with – it's a long story – focus on the problem).
So from the above we get the below tree:



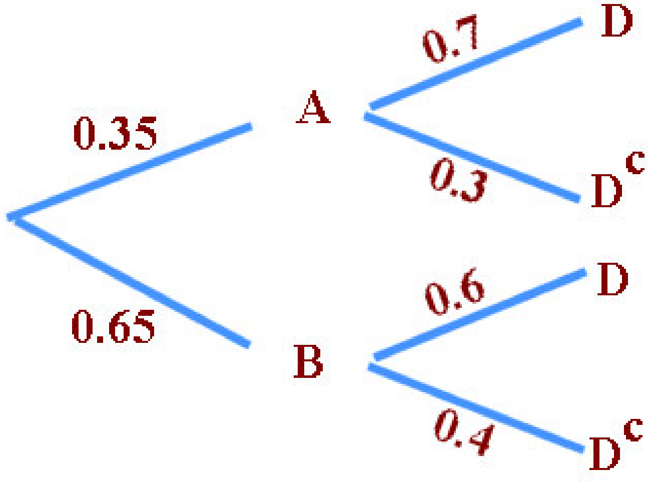
Now because we are doing missionary work in Africa we are only able to find out that Tom does indeed see Kelly. Now we really wanted Bob to go out with Linda. So having some time to kill we want to figure out what the probability is that Bob went out with Linda, given that Tom went out with Kelly.

So the path we are interested in is the top path, it has a probability of 0.4*0.3 = 0.12.
There are two other paths that lead to Tom seeing Kelly.
The middle of these two paths has a probability of 0.4*0.4 = 0.16.
And the bottom of these two paths has a probability of 0.2*0.5 = 0.1.

So
P(Bob seeing Linda | Tom saw Kelly)
= (prob. of top path ending with Kelly) / (sum of probs. **all 3** paths ending with Tom and Kelly)
= 0.12 / (0.12 + 0.16 + 0.1)
≈ 0.3157894

Anyway what you should see is that while the formula reads and looks complicated, solving the problems is not. Look at the examples in the book also.

Problem 8:
The below diagram represents a two-stage experiment. Use it to answer questions (a) through (c).



8a. P(A) * P(D | A)

From the diagram P(A) = 0.35 and P(D |A) = 0.7

So the answer is 0.35 * 0.7 = **0.245**

8b. P(B) * P(D | B)

From the diagram P(B) = 0.65 and P(D |B) = 0.6

So the answer is 0.65 * 0.6 = **0.39**

8c. P(A | D)

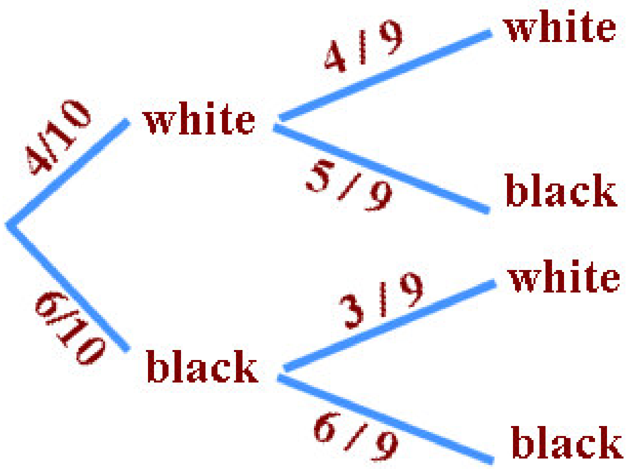
This by Bayes Theorem is

[answer (a)] / [answer (a) + answer (b)] = 0.245 / (0.245+0.39) ≈ 0.38582677

Problems 15 – 18:
Urn A contains 4 white and 6 black balls. Urn B contains 3 white and 5 black balls. One ball is drawn from urn A and then transferred to urn B. A ball is then drawn from urn B.

First set up the tree (which is problem 15). Notice that if a white ball is selected from A then when the selection is made from urn B there will be 4 white and 5 black balls. However if a black ball is selected from A then there will be 3 white and 6 black balls in urn B.

Problem 15:
Represent the probabilities associated with this two-stage experiment in the form of a tree diagram.



Problem 16:
What is the probability that the transferred ball (the one selected from A) was white given that the second ball drawn was white?

So the prob. of our desired path (white-white) is 4/10 * 4/9 = 16 / 90.
The prob. of the other possible path ending in white (black-white) is 6/10*3/9 = 18 / 90.
Thus

p(first white | second was white) = (16/90) / (16/90+18/90) = 16/34 = **8 / 17**

Problem 17:
What is the probability that the transferred ball (the one selected from A) was black given that the second ball drawn was white?

So the prob. of our desired path (black-white) is 6/10 * 3/9 = 18 / 90.
The prob. of the other possible path ending in white (white-white) is 4/10*4/9 = 16 / 90.
Thus

p(first black | second was white) = (18/90) / (18/90 + 16/90) = 18/34 = **9 / 17**

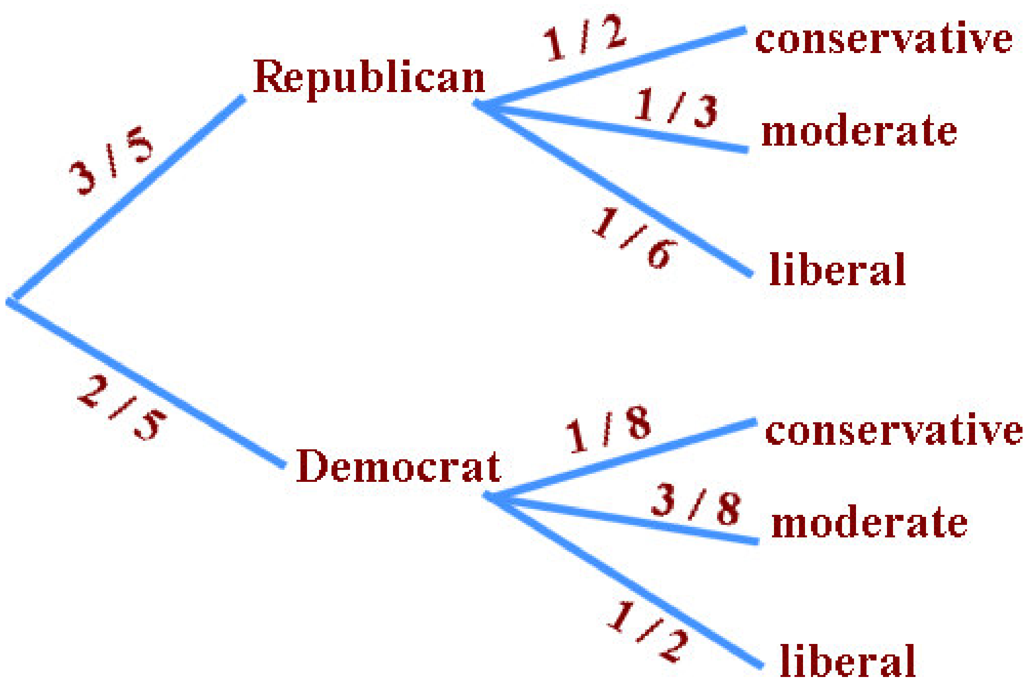
Problem 18:
What is the probability that the transferred ball (the one selected from A) was black given that the second ball drawn was black?

So the prob. of our desired path (black-black) is 6/10 * 6/9 = 36 / 90.
The prob. of the other possible path ending in black (white-black) is 4/10*5/9 = 20 / 90.
Thus

p(first black | second was white) = (36/90) / (36/90 + 20/90) = 36/56 = **9 / 14**

Problem 32:
In a past presidential election, it was estimated that the probability the Republican candidate would be elected was 3/5 and therefore the probability the Democratic candidate would be elected was 2/5 (the Independent candidates were given little chance of being elected). It was also estimated that if the Republican candidate was elected the probability that a conservative, moderate or liberal judge would be appointed to the Supreme Court was ½, 1/3 and 1/6 respectively. if the Democratic candidate was elected the probabilities that a conservative, moderate or liberal judge would be appointed to the Supreme Court would be 1/8, 3/8 and 1/2 respectively. A conservative judge was appointed to the Supreme Court during the presidential term. What is the probability that the Democratic candidate was elected?

Make up the tree diagram, from that it should be pretty easy to get an answer:



So the probability that the Republican was elected and a conservative was appointed is

$p(\text{Republican} \cap \text{conservative}) = p(\text{Rep})*p(\text{con}|\text{Rep}) = 3/5 * 1/2 = \underline{3 / 10}$.

The prob. that the Democrat was elected and a conservative judge was appointed is

$p(\text{Democrat} \cap \text{conservative}) = p(\text{Dem})*p(\text{con}|\text{Dem})2/5 * 1/8 = \underline{2 / 40}$.

By Bayes Thm then prob(Democrat was elected | conserv. judge appointed) is

$p(\text{ Democrat} | \text{conservative}) = (2/40) / (2/40 + 3/10) = \mathbf{1 / 7}$.