Finite Math Section 8_3
Solutions and Hints

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for the book:

Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 8.3:

There are three things about notation you need to memorize:

Let X be a random variable then:

1. Expected Value of X = E(X) = μ
2. Variance of X = Var(x) = σ^2.
3. Standard Deviation of X = σ

Of course you still must memorize the formulas. Notice that standard deviation is just the square root of variance, so you really only need to memorize 2 formulas =)

Given that X has the probability distribution:

x	x_1	x_2	x_3	…	x_n
P(X = x)	p_1	p_2	p_3	…	p_n

1. $\mu = E(X) = x_1 * p_1 + x_2 * p_2 + x_3 * p_3 + \ldots + x_n * p_n$.

2. $\sigma^2 = Var(X) = p_1(x_1 - \mu)^2 + p_2(x_2 - \mu)^2 + \ldots + p_n(x_n - \mu)^2$.
   Variance is also expressed as: $Var(X) = E(X^2) - \mu^2$.

You must also memorize Chebychev's Equation:

$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq (1 - 1/k^2)$

This is used when you want a lower and upper bound for your X value.
You will most often be given μ and σ and will need to find k.

Problem 16:
The distribution of the number of chocolate chips (x) in a cookie is shown in the following table. Find the mean and the variance of the number of chocolate chips in a cookie.

x	0	1	2	3	4	5	6	7	8
P(x)	0.01	0.03	0.05	0.11	0.13	0.24	0.22	0.16	0.05

E(X) = 0*.01 + 1*0.03 + 2*0.05 + 3*0.11 + 4*0.13 + 5*0.24 + 6*0.22 + 7*0.16 + 8*0.05
μ = E(X) = **5.02**

Var(X) = $0.01*(0-5.02)^2 + 0.03*(1-5.02)^2 + 0.05*(2-5.02)^2 + 0.11*(3-5.02)^2 + 0.13*(4-5.02)^2 + 0.24*(5-5.02)^2 + 0.22*(6-5.02)^2 + 0.16*(7-5.02)^2 + 0.05*(8-5.02)^2$.

Var(X) = 0.252004 + 0.484812 + 0.45602 + 0.448844 + 0.135252 + 0.000096 + 0.211288 + 0.627264 + 0.44402

Var(X) = **3.0596**

Problem 28:
A Christmas tree light has an expected life of 200 hours and a standard deviation of 2 hours.

28a. Estimate the probability that one of these Christmas tree lights will last between 190 and 210 hours.

So the question in equation form is to find: $P(190 \le X \le 210)$

Use Chebychev's equation: $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge (1 - 1/k^2)$
Notice we are given the lower and upper bounds of 190 and 210.
We are given that $\mu = 200$ and $\sigma = 2$. So we say:

$\mu - k\sigma = 190$	and	$\mu + k\sigma = 210$
200- k*2 = 190	and	200 +k*2 = 210
10 = k*2	and	k*2 = 10
5 = k	and	k = 5

Notice we could have just solved one of the above equations, however, its always good to see you get the same result with whichever one you choose.

Putting stuff together we see:
$P(190 \le X \le 210) \ge (1 - 1/k^2) \rightarrow P(190 \le X \le 210) = 1 - 1/5^2 = 1 - 1/25 =$ **24 / 25**

28b. Suppose 150,000 o these Christmas tree lights are used by a large city as part of its Christmas decorations. Estimate the number of lights that will require replacement between 180 and 220 hours of use.

Notice we first need to find the probability that a single light bulb will need replaced between 180 and 220 hours of use. This works the same as we did in part (a).

Notice we will first find the probability that a single light bulb will NOT need replaced:
So the question in equation form is to find: $P(180 \le X \le 220)$

Use Chebychev's equation: $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge (1 - 1/k^2)$
Notice we are given the lower and upper bounds of 180 and 220.
We are given that $\mu = 200$ and $\sigma = 2$. So we say:

$\mu - k\sigma = 180$	and	$\mu + k\sigma = 220$
$200 - k*2 = 180$	and	$200 + k*2 = 220$
$20 = k*2$	and	$k*2 = 20$
$10 = k$	and	$k = 10$

Notice we could have just solved one of the above equations, however, its always good to see you get the same result with whichever one you choose.

Putting stuff together we see:
$P(180 \le X \le 220) \ge (1 - 1/k^2) \rightarrow P(180 \le X \le 220) = 1 - 1/10^2 = 90/100 = 9/10$.

So the probability that a single light bulb will NOT need replaced between 180 and 220 hours is 9 / 10. Thus the probability that it will need replaced is 1 – 9/10 = <u>1/10</u>.

We however are asked how many of 150,000 light bulbs will need replaced. Since each has a 1/10 of failure (on average) we simply say that
on average 1/10*150000 = **15,000 will need replaced**.