

Finite Math Section 8_4

Solutions and Hints

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for the book:
Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 8.4:

This section focuses on Binomial (Bernoulli) Experiments. Most all of this stuff in this chapter needs memorized. It is summarized below:

A **binomial experiment** has the following properties:

1. The number of trials in the experiment is fixed.
2. There are TWO outcomes of the experiment (success and failure)
3. The probability of success in each trial is the same.
4. The trials are independent of each other.

Generally **p = the probability of success** and **q = the probability of failure**

As those are the only two results $p + q = 1$.

A nice formula to remember is: in a binomial experiment the probability of x successes in n trials can be found by: **$C(n, x)p^xq^{n-x}$** . KNOW THIS ONE!

Some other formulas to remember if X is a Binomial random variable:

The mean = $E(X) = \mu = n \cdot p$

The variance = $\text{Var}(X) = \sigma^2 = n \cdot p \cdot q$

The standard deviation = $\sigma = (n \cdot p \cdot q)^{1/2}$.

Problem 17:

A fair die is cast four times. Calculate the probability of obtaining exactly two 6's.

This is straight formula for a binomial experiment: $C(n, x)p^xq^{n-x}$.

With $n = 4,$
 $x = 2$
 $p = 1/6$
 $q = 5/6$

$$C(4, 2) \cdot (1/6)^2 \cdot (5/6)^{4-2} = \mathbf{25 / 216}.$$

Problem 20:

If the probability of that a certain tennis player will serve an ace is $1/4$, what is the probability that he will serve exactly two aces out of five?

This is straight formula for a binomial experiment: $C(n, x)p^xq^{n-x}$.

With $n = 5,$
 $x = 2$
 $p = 1/4$
 $q = 3/4$

$$C(5, 2) \cdot (1/4)^2 \cdot (3/4)^{5-2} = \mathbf{135 / 512}.$$

Problem 24:

From experience the manager of Kramer's Book Mart knows that 40% of the people who are browsing in the store will make a purchase. What is the probability that among ten people who are browsing in the store AT LEAST three will make a purchase.

Recall $P(\text{at least } 3) = P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$

And that is a lot of stuff to calculate, however we might save some time by remembering

$$\begin{aligned} P(\text{at least } 3) &= 1 - P(2 \text{ or less}) \\ &= 1 - P(0) - P(1) - P(2) \end{aligned}$$

$$P(0) = C(10, 0) \cdot (0.4)^0 \cdot (0.6)^{10} = 59049 / 9765625$$

$$P(1) = C(10, 1) \cdot (0.4)^1 \cdot (0.6)^{10-1} = 78732 / 1953125$$

$$P(2) = C(10, 2) \cdot (0.4)^2 \cdot (0.6)^{10-2} = 236196 / 1953125$$

$$P(\text{at least } 3) = 1 - 59049 / 9765625 - 78732 / 1953125 - 236196 / 1953125$$

$$= 1 - (1633689 / 9765625)$$

$$= \mathbf{8131936 / 9765625}$$

$$\approx 0.83271$$

Problem 32:

A psychology quiz consists of ten true-or-false questions. If a student knows the correct answer to six of the questions but determines the answers to the remaining questions by flipping a coin, what is the probability that she will obtain a score of at least 90% ?

First notice that 60% is guaranteed. So of the four remaining she must guess AT LEAST three correctly to obtain at least 90%. The probability of her guessing any single question correctly is $\frac{1}{2}$.

Since we need at least 3 guesses correct that means either exactly 3 or exactly 4 are correct. We will first calculate the probability of exactly 3 using the equation:

$$C(n, x)p^xq^{n-x}.$$

With $n = 4,$

$$x = 3$$

$$p = 1/2$$

$$q = 1/2$$

$$C(4, 3) \cdot (1/2)^3 \cdot (1/2)^{4-3} = \underline{1/4} = \text{probability of guessing exactly 3 correct.}$$

Now we find the probability of getting exactly 4 correct the same way:

$$C(n, x)p^xq^{n-x}.$$

With $n = 4,$

$$x = 4$$

$$p = 1/2$$

$$q = 1/2$$

$$C(4, 4) \cdot (1/2)^4 \cdot (1/2)^{4-4} = \underline{1/16} = \text{probability of guessing exactly 4 correct.}$$

So the chances of guessing 3 or more questions correctly and thus obtaining a score of at least 90% is $1/4 + 1/16 = \mathbf{5 / 16}$.