Finite Math Section 8_5
Solutions and Hints

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for the book:

Finite Mathematics, 7th Edition
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

Section 8.5:

The key to this section is figuring out how to read the graph of the normal distribution curve. it is not that hard. Look at the examples and listen to your instructor.

The other thing to remember is that the **standard normal curve** (the one you see the picture of most often) has a mean, $\mu = 0$. And a standard deviation, $\sigma = 1$.

The one equation you need to remember is that if you are given that X is a NORMAL random variable with some mean μ and some standard deviation σ you need to make one small change and say $Z = (X - \mu) / \sigma$. You then plug whatever range you are asked about for X into that equation to get the value of Z. You then look up the value(s) of Z in Table 2 of Appendix C in your book (pages 601 and 602)

There is should be a web page available to show you how to read the table using some simple examples. Probably available from the same site you grabbed this page =).

Most all of the problems in this section are simply looking stuff up in the table. You should be able to do these look up operations quickly and easily. The skill will be necessary to answer at least one test question.

<u>Problem 16</u>:

Let Z be the standard normal variable. Find the values of z if z satisfies:

16a. $P(Z > z) = 0.9678$

This one all you do is scan Table 2, Appendix C (page 601, 602) for the value 0.9678
You will see that it falls under the 0.05 heading and is in the row for $z = 1.8$.

So the answer is $\mathbf{z = 1.85}$

16b. $P(-z < Z < z) = 0.8354$

This is a bit trickier, for a full explanation see example 2, part c on page 492 for a more complete explanation.

The basic idea is this (and if you look at the picture this makes sense):

Eq. 1: $P(-z < Z < z) = 2 * P(0 < Z < z)$

And

Eq. 2: $P(0 < Z < z) = P(Z < z) - ½$ ← *put this into Eq. 1*

So

$P(-z < Z < z) = 2*P(0 < Z < z) \rightarrow P(-z < Z < z) = 2*(P(Z < z) - ½)$ *(solve for P(Z <z)*

Solving for $P(Z < z)$ *(because we can look up the solution to that in the table)* we get:

$P(-z < Z < z) = 2*(P(Z < z) - ½)$ → $½ * P(-z < Z < z) = P(Z < z) - ½$

→ $½ * P(-z < Z < z) + ½ = P(Z < z)$

So now we know that

$P(Z < z) = ½*P(-z < Z < z) + ½$ (and we were given $P(-z < Z < z) = 0.8354$)

Thus

$P(Z < z) = ½*0.8354 + ½$

$P(Z < z) = 0.9177$

So we look up 0.9177 in the chart and find it under then 0.09 column in the $z = 1.3$ row.

So the answer is $\mathbf{z = 1.39}$.

You can check this using the table.

$P(Z < -1.39) = 0.0823$

$P(Z < 1.39) = 0.9177$

and $0.9177 - 0.0823 = 0.8354$ which is what we were told $P(-z , Z < z)$ was.

<u>Problem 18</u>:
Suppose X is a normal random variable with $\mu = 380$ and $\sigma = 20$.

18a. Find the value of P(X < 405)
First we need to standardize our random variable so we say:
$Z = (X - \mu) / \sigma$
$Z = (X - 380) / 20$

So the question becomes find the value of $P(Z < (405 - 380) / 20)$
Or rather find $P(Z < 1.25)$
Using the table we find the answer to be: **0.8944**

18b. Find the value of P(X > 400)
Notice we will first find $P(X < 400)$ and then say $P(X > 400) = 1 - P(X < 400)$

Again we must standardize our random variable:
$Z = (X - \mu) / \sigma$
$Z = (X - 380) / 20$

So the question becomes find the value of $P(Z < (400 - 380) / 20)$
Or rather find $P(Z < 1) = P(X < 400)$
Using the table we find this answer to be: <u>0.8413 = P(X < 400)</u>

Now we use $P(X > 400) = 1 - P(X < 400) = 1 - 0.8413$
And we get the answer
$P(X > 400) =$ **0.1587**

18c. Find the value of P(400 < X < 430)
Notice in part (b) we solved for $P(X > 400)$ so all we need to do is solve for $P(X > 430)$ and declare our answer to be:
$P(400 < X < 430) = P(X > 400) - P(X > 430)$

Again we must standardize our random variable: $Z = (X - 380) / 20$

So the question becomes find the value of $P(Z < (430 - 380) / 20)$
Or rather find $P(Z < 2.5) = P(X < 430)$
Using the table we find this answer to be: <u>0.9938 = P(X < 430)</u>
So $P(X > 430) = 1 - P(X < 430) =$ <u>0.0062</u>
And from (b) we know $P(X > 400) =$ <u>0.1587</u>

Using $P(400 < X < 430) = P(X > 400) - P(X > 430)$
We get $P(400 < X < 430) = 0.1587 - 0.0062 =$ **0.1525**