Finite Math Section 8_6
Solutions and Hints

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for the book:

<u>Finite Mathematics, 7th Edition</u>
by S. T. Tan.

DO NOT PRINT THIS OUT AND TURN IT IN !!!!!!!!
This is designed to assist you in the event you get stuck.
If you do not do the work you will NOT pass the tests.

<u>Section 8.6:</u>

For the most part this entire section is dedicated to applications of 8.5.

The only trick they introduce here is a way to solve Binomial Distributions by approximating them with a normal distribution. In sum it goes as follows:

Say you are given a binomial experiment (say flipping coins) and the number of trials is large (like more than 20) and the number of successes is more than 10. You could solve the problem using the $P(X = x) = C(n, x)p^x q^{n-x}$ equation for each success case. However if you have more than 10 possible success cases this could take a great deal of time to work out (which is bad when you are taking a test). So under those conditions the book shows you that you can call the Binomial Experiment a Normal Distribution with average mean = $E(X) = \mu = np$ and standard deviation = $\sigma = (npq)^{1/2}$. And you then do some simple tricks.

First let X denote the number of success in your original binomial experiment.
Assume you want $P(X > x)$ or $P(X < x)$.
You then attempt to solve $P(Y > x+0.5)$ or $P(Y < x-0.5)$ *Don't ask why – it just works.*
You then standardize the problem for a normal distribution and solve using the normal distribution tables:
$P(Z > [(x+0.5) – np) / (npq)^{1/2}])$ *if your original problem was P(X > x)*
or $P(Z < [(x-0.5) – np) / (npq)^{1/2}])$ *if your original problem was P(X < x)*

There is also a small note that p must not be "near 0" say $p \geq 0.05$ should work.

<u>Problem 3</u>:
TKK Products manufactures electronic light bulbs in the 50, 60, 75 and 100-watt range. Lab tests show that the lives of these light bulbs are normally distributed with a mean of 750 hours and a standard deviation of 75 hours.

3a. What is the probability that a TKK light bulb selected at random will burn for more than 900 hours?
Notice that:
$\mu = 750$
$\sigma = 75$

Normalize this distribution by:
Let $Z = (X - \mu) / \sigma$
So
$P(X > 900) \quad = P(Z > (900 - 750) / 75)$
$\quad = P(Z > 2)$
$\quad = P(Z < -2)$ *(or if you prefer = 1 – P(Z < 2))*
$\quad =$ **0.0228** *(or 1 – 0.9772= 0.228)*

3b. What is the probability that a TKK light bulb selected at random will burn for less than 600 hours?
Notice that: $\mu = 750$ and $\sigma = 75$
Normalize this distribution by:
Let $Z = (X - \mu) / \sigma$
So
$P(X < 600) \quad = P(Z < (600 - 750) / 75)$
$\quad = P(Z < -2)$
$\quad =$ **0.0228**

3c. What is the probability that a TKK light bulb selected at random will burn between 750 and 900 hours?
Notice $P(750 < X < 900) = P(X < 900) - P(X < 750)$

From part (a) $P(X > 900) = 0.0228$, so $P(X < 900) = 1 - 0.0228 = \underline{0.9772}$.

Find $P(X < 750)$:

Notice that: $\mu = 750$ and $\sigma = 75$
Normalize this distribution by:
Let $Z = (X - \mu) / \sigma$
So
$P(X < 750) \quad = P(Z < (750 - 750) / 75)$
$\quad = P(Z < 0)$
$\quad = \underline{0.5000}$

$P(750 < X < 900) = P(X < 900) - P(X < 750) = 0.9772 - 0.5000 =$ **0.4772**

3d. What is the probability that a TKK light bulb selected at random will burn between 600 and 800 hours?

Notice $P(600 < X < 800) = P(X < 800) – P(X < 600)$

From part (b) <u>$P(X < 600) = 0.0228$</u>.

Find $P(X < 800)$:

Notice that: $\mu = 750$ and $\sigma = 75$

Normalize this distribution by:

Let $Z = (X – \mu) / \sigma$

So

$$\begin{aligned} P(X < 800) &= P(Z < (800 – 750) / 75) \\ &= P(Z < 2/3) \approx P(Z < 0.67) \\ &\approx \underline{0.7486} \end{aligned}$$

$P(600 < X < 800) = P(X < 800) – P(X < 600) = 0.7486 – 0.0228 =$ **0.7258**

<u>Problem 4</u>:

On average, a student takes 100 words/minute midway through an advanced court reporting course at the American Institute of Court Reporting. Assuming that the dictation speeds of the students are normally distributed and that the standard deviation is 20 words/minute:

4a. What is the probability that a student randomly selected from the course can take dictation at a speed of more than 120 words/minute?

Notice $\mu = 100$ and $\sigma = 20$

Let $Z = (X - \mu) / \sigma$, so

$$\begin{aligned} P(X > 120) &= P(Z > (120 – 100) / 20) \\ &= P(Z > 1) \\ &= P(Z < -1) \quad \textit{(or } 1 – P(Z < 1)\textit{)} \\ &= \mathbf{0.1587} \quad \textit{(or } 1 – 0.8413 = 0.1587\textit{)} \end{aligned}$$

4b. What is the probability that a student randomly selected from the course can take dictation at a speed between 80 and 120 words/minute?

Notice $P(80 < X < 120) = P(X < 120) – P(X < 80)$

From part (a) $P(X > 120) = 0.1587$ so $P(X < 120 = 1 – 0.1587 =$ <u>0.8413</u>
and from part (c) <u>$p(X < 80) = 0.1587$</u>. So:

$P(80 < X < 120) = P(X < 120) – P(X < 80) = 0.8413 – 0.1587 =$ **0.6826**

4c. What is the probability that a student randomly selected from the course can take dictation at a speed of less than 80 words/minute?

Notice $\mu = 100$ and $\sigma = 20$

Let $Z = (X - \mu) / \sigma$, so

$P(X < 80) = P(Z < (80 - 100) / 20)$

$= P(Z < -1)$

$=$ **0.1587**

Problem 10:

The scores on an Economics examination are normally distributed with a mean of 72 and a standard deviation of 16. If the instructor assigns a grade of A to 10% of the class, what is the lowest score a student may have and still obtain an A?

We need to find x (the score) such that $P(X < x) = 0.9$
(or rather the score which 90% of the class scored below)

Notice $\mu = 72$ and $\sigma = 16$.
Let $Z = (X - \mu) / \sigma$

Now we need to find x such that:

$P(Z < (x - 72) / 16) = 0.9$

We will first find z such that $P(Z < z) = 0.9$:

So look up 0.9 in Table 2 of Appendix C (page 601, 602) and we see
$z \approx 1.28$ (since 0.8997 is closer to 0.9 than 0.9015)

Notice we could get a better estimate for z – ask your instructor how.

Now we solve for x by saying:

$z = (x - 72) / 16$
$1.28 = (x - 72) / 16$
$20.48 = x - 72$
$92.48 = x$

So we say,
the lowest score for a student to receive and still make an A would be around **92.48**.

Problem 18:
Jorge sells magazine subscriptions over the phone. He estimates that the probability of his making a sale with each attempt is 0.12. What is the probability of Jorge making more than 10 sales if he makes 80 calls?

Notice the instructions state that this is to be solved by approximating the binomial experiment with a normal distribution.

Notice $n = 80$, $p = 0.12$, $q = 0.88$
And we want X = number of sales > 10

So we let
$\mu = np = 80 * 0.12 = 9.6$
$\sigma = (npq)^{1/2} = (80*0.12*0.88)^{1/2} = 2.90654434$

We want to solve $P(X > 10)$ which we approximate by saying $P(Y > 10.5)$.
We now let $Z = (Y - \mu) / \sigma$
So $P(Y > 10.5)$ $= P(Z > (10.5 – 9.6) / 2.90654434)$
$= P(Z > 0.31)$
$= P(Z < -0.31)$ *or if you prefer* $= 1 – P(Z < 0.31)$
$= 0.3783$ *or* $1 – 0.6217 = 0.3783$

Thus $P(X > 10) \approx$ **0.3783**.