Simplex Method
for standard linear programming problems

A Standard Maximization Problem has the following description:

1. The objective function is to be maximized.
2. All the variables (x, y, z, …) are non-negative.
3. All other constraints have the form $Ax + By + Cz + \ldots \leq N$ (and not $\geq$).

The below is NOT as difficult as it might seem.

To solve a Standard Maximization Problem:

1. **Rewrite every inequality constraint as an equality using slack variables.**
   Notice that you will use a different slack variable for each inequality.
   For example if you had
   Eq 1: $x + y \leq 50$
   Eq 2: $2x - y \leq 30$
   You would introduce two slack variables: u and v. Specifically:
   Eq 1 would become: $x + y + u = 50$ and
   Eq 2 would become: $2x - y + v = 30$

2. **Rewrite the objective function $P = ax + by + cz + \ldots$ to be in the form of $-ax - by - cz \ldots + P = 0$**
   (i.e. subtract all the variables over to the same side as P)

3. **Write the augmented matrix of the above system of equations.**
   This is called the INITIAL SIMPLEX TABLEAU
   Notice the rewritten objective function goes in the last row.

4. **Choose the pivot column or terminate the algorithm**:
   Look at the last **row**.
   a. If the last row row contains at least one negative number (other than the entry furthest to the right)
      Then put an arrow under the most negative entry in the last row.
      The column this arrowed entry is in, is the pivot column.

   b. If all the entries (except the rightmost column) of the last row are positive or zero then you can read out the solution as follows:

      i. If a column for a variable has more than one non-zero entry then set that variable equal to zero.
      ii. If a column for a variable has exactly one non-zero entry (whose value should be 1) then the variable is set equal to the constant (the value in the rightmost column) in the row containing the non-zero entry.

5. **Determine the pivot row**:
   For each positive element, *a*, in the pivot column:
   a. Form the quotient *c* / *a*, where *c* is the constant (the number in the rightmost column) of the row *a* appears in.
   b. Determine which of these quotients is the smallest non-negative number.
   c. The row from which this smallest quotient came from is the pivot row. Mark it by circling the associated element *a*.

6. **Determine the pivot element and make its value one**.
   The element you circled in 5(c) is the pivot element.
   If its value is NOT one, then divide all the elements in its row by its value (thus its value becomes one).

7. **Zero out all the numbers above and below the pivot element.**
   Use row operations to add/subtract the pivot element's row to all the other rows so the pivot element is the only element in its column with a non-zero entry.

8. **Repeat – go back to steps 4 to 7 until you find a solution**.