Practice Problems for 142 Final Exam:

1. Find the domain of $f(x) = \dfrac{\sqrt{x-4}}{x^2 - 5x + 6}$

2. Find the equilibrium point if the demand equation is $p = .1x + 2$ and the supply equation is $p = .2x + 1$.

3. If $f(x) = 3x^2 - 5x + 2$, find $\dfrac{f(x+h) - f(x)}{h}$ and simplify completely.

4. You make a 100 on your first calculus test in college and your parents are so excited that they give you \$1000. There is a catch though (isn't there always), - you can't spend any of the money until you graduate in 6 years (yes...it takes you 6 years). So you deposit the money into an account that pays $7\frac{3}{4}\%$ interest compounded weekly. How much money do you have after the 6 years?

5. Solve for x:

   (a) $4^{3x} = 8^{-x+5}$ (b) $6 = 7 \cdot 10^{3x}$ (c) $e^{4\ln 3} = x$ (d) $5\ln(2x+8) - 4 = 0$

   (e) $3 \cdot \log_7 7^4 = x$ (f) $100 = 25(1.5)^t$ (g) $7(3^x) = 4(5^x)$ (h) $3^{x-2} = 4^{5+x}$

   (i) $\log_2(\log 4x) = 1$ (j) $25^{3x+4} = \dfrac{5^x}{25}$

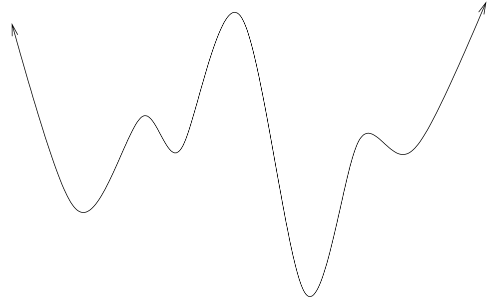
6. If the graph of $y = x^2$ is shifted up three units, to the right 4 units, and reflected about the x-axis, what is the equation representing the resulting graph?

7. When you jump out of an airplane and your parachute fails to open, your downward velocity (in meters/second), t seconds after the jump, is approximated by

$$v(t) = 49(1 - (0.8187)^t)$$

   How far did you fall during the first five seconds?

8. For the polynomial, $f(x)$, graphed to the right,



   (a) What is the minimum degree possible for $f(x)$?
   (b) Is $f(x)$ positive or negative?
   (c) Is $f(x)$ odd or even?

year	1986	1987	1988	1989	1991	1992
population (in 1000's of people)	2	2.4	2.5	2.8	3.1	3.2

9. The table above shows the population, in thousands of people, of a small town, from 1986-1992. Find the average rate of change of people/year between 1987 and 1991.

10. Proud parents of a newborn child establish an educational fund for the child by depositing \$1500 in an account that pays 7.65% interest compounded monthly.

    (a) What will the amount be in 18 years?
    (b) If the parents want \$20,000 after 18 years, how much should they deposit initially?
    (c) If \$3000 is deposited initially, how long will it take until there is \$10,000 in the account?

11. A florist collected the following data regarding the sales of roses on Valentine's Day: (price is the dependent variable)

Price of 1 dozen roses (in dollars)	10	15	20	25	30	35
# dozens sold on Valentine's Day	190	145	110	86	65	52

    (a) Determine the equation that best fits the data above.
    (b) Predict the number of dozens the florist will sell if she fixes her price at \$40/dozen.
    (c) Estimate what the florist may have been charging if she sold 200 dozens of roses.

12. Find the lower estimate of $\int_1^4 \frac{2}{x}dx$ using Riemann Sum with n=6.

13. Find the area between $y = -\ln(x+3)$ and $-x^2 - 3x + 2$.

14. If $K = ab\left(c + \frac{1}{m}\right)^j$ find:

- $\frac{dK}{dj}$
- $\frac{dK}{dm}$

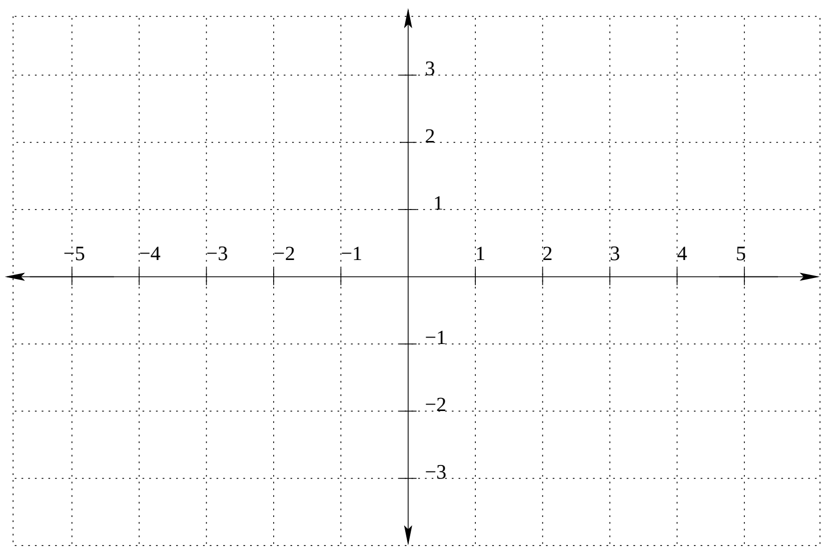
15. Suppose $C(q)$ represents how much it costs to produce thing-a-ma-bobs. The fixed costs for producing thing-a-ma-bobs is \$20,000. The marginal cost function is given by $C'(q) = 0.005q^2 - q + 56$. Find the total cost to produce 50 thing-a-ma-bobs.

16. Evaluate the definate integral $\displaystyle\int_2^5 \frac{3}{\sqrt{4+x^2}}dx$.

17. As soon as you bought your Chemistry book, dust started collecting on it at a rate given by $r(t) = \dfrac{2t}{2+3t^2}$ in ounces of dust per day. How much dust did your Chemistry book collect during the first 3 weeks of school (assuming you bought the book the right when school started)?

18. Draw a possible graph of $f(x)$ given the following information about its derivative:

    - x-intercepts at -1
    - vertical asymptote at x=0
    - $\lim\limits_{x\to-\infty} f(x) = 0$
    - $\lim\limits_{x\to\infty} f(x) = 0$
    - $f'(-2) = 0$
    - $f'(x) > 0$ on $(-2, 0)$ and $(0, \infty)$
    - $f'(x) < 0$ on $(-\infty, -2)$
    - $f''(x) < 0$ on $(-\infty, -3)$ and $(0, \infty)$
    - $f''(x) > 0$ on $(-3, 0)$

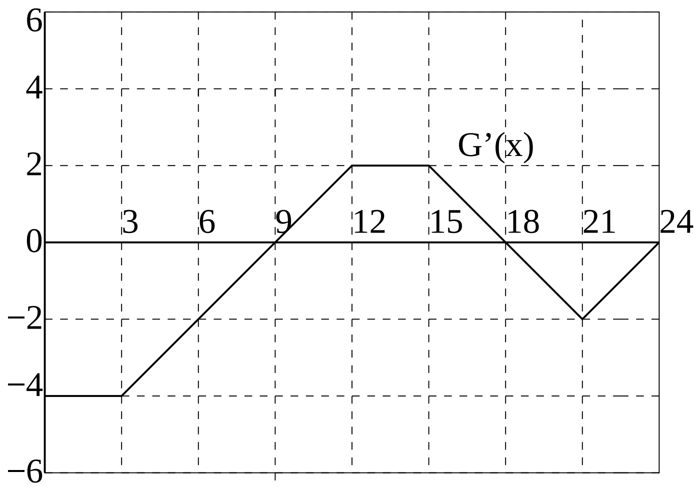


Time (weeks)	0	5	10	15	20
Rate of sales (games per week)	0	585	892	1875	2350

19. The rate of sales (in games per week) of a new video game is shown in the table above. Assuming that the rate of sales increased throughout the 20-week period, give upper and lower estimates of the total number of games sold during this period.

20. Find the equation of the tangent line of $f(x) = 9.354(.765)^x$ at $x = 2$.

21. Find all absolute extrema for $f(x) = -3x^4 + 6x^2 - 5$ on the interval $(-\infty, \infty)$.



22. Using the figure above,

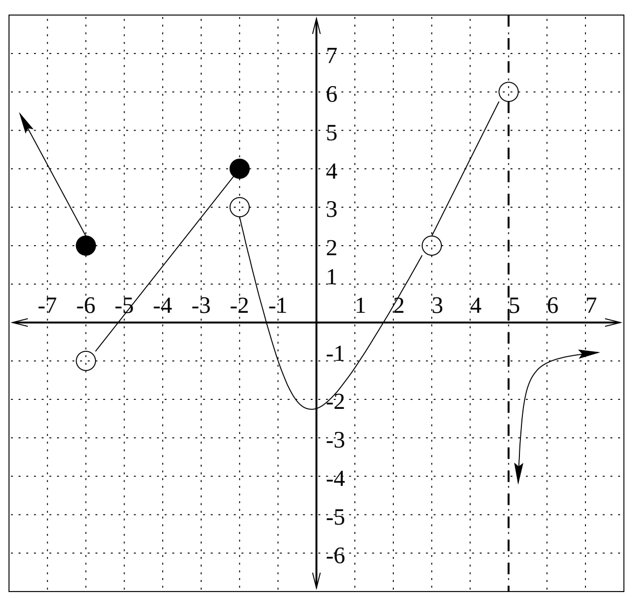
    - Estimate the total area between the curve and the x-axis from x=0 to x=24.
    - Estimate $\int_0^{24} G(x)dx$.

23. Evaluate the following limits:

    (a) $\lim_{x\to 0} \frac{x}{x^2+x+1}$ (b) $\lim_{x\to 0} \frac{x}{x^2+x}$ (c) $\lim_{x\to -1} \frac{x^2-1}{x+1}$ (d) $\lim_{x\to 2} \frac{x-1}{(x-2)^2}$

    (e) $\lim_{x\to \infty} \frac{x}{x^2+x+1}$ (f) $\lim_{x\to -\infty} \frac{e^x}{2x-1}$ (g) $\lim_{x\to \infty} \frac{6x+3x^3}{4x^2+6x^3-2}$

24. Use the graph of $f(x)$ to the right to find the limits:



    (a) $\lim_{x\to -6^+} f(x)$

    (b) $\lim_{x\to -6^-} f(x)$

    (c) $\lim_{x\to -6} f(x)$

    (d) $\lim_{x\to 3} f(x)$

    (e) $\lim_{x\to 1} f(x)$

    (f) $\lim_{x\to 5} f(x)$

    (g) Estimate the intervals for which $f'(x)$ is negative.

    (h) Is $f''(x)$ postive or negative around x=0?

Find the derivatives of each of the following:

25. $f(x) = x^\pi + 4 \cdot 2^x + e^{\sqrt{x}} + \dfrac{3}{x}$

26. $y = 12x - \ln(e^{3x} + 3ex^4)$

27. $h(x) = e + \dfrac{x^2 + \sqrt[3]{x} + x}{5 + 2^{4x}}$

28. $y = \sqrt[3]{3 + 3x + \dfrac{1}{x^3}}$

29. $g(x) = 3\ln(e^{-x} - x) + 12(3x^2 + 4x)^5$

30. $C(q) = \dfrac{\frac{4}{q^2} + e^{q^2+\pi}}{(2q^5 - 7e^{3q})^{\frac{3}{5}}}$

31. The table below gives the population of the Northeast in millions for some selected years.

Year	1790	1830	1870	1910	1950	1990
Population (in millions)	2.0	5.5	12.3	25.9	39.5	50.8

(a) Let $x = 0$ represent 1790. Find the best-fitting logistic equation?

(b) What tends to be the limiting (maximum) population for the northeast?

(c) During what year did the population reach 10 million people?

32. A government bond will pay \$10,000 at maturity 20 years from now. How much is the bond worth today if the interest rate is 5.4% compounded daily?

33. Evaluate $\displaystyle\int_a^{2a} (3x^2 - 7)\,dx$ and simplify completely.

34. Using the equation for elasticity of demand, $E = -\frac{p}{x} \cdot \frac{dx}{dp}$, find the elasticity of $x = 20 - \frac{5}{p^2}$ at p=.02.

35. Based upon your answer for the previous problem, is the demand:
(a) elastic (b) inelastic (c) of unit elasticity

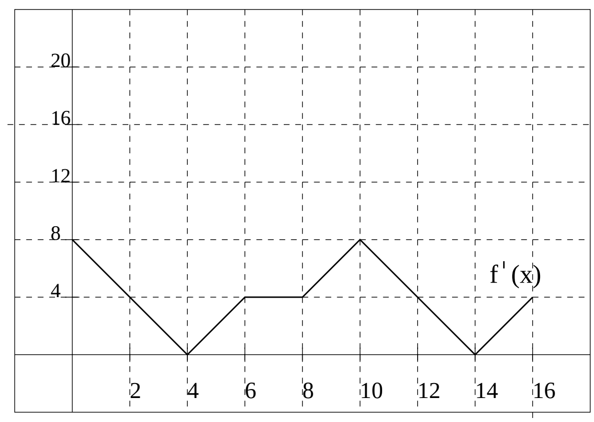
For the following functions, find the critical values, use first derivative test to find the relative extremum and intervals of increasing/decreasing, and the 2nd derivative test to find the intervals of concavity.

36. $y = \frac{1}{3}x^3 + 3x^2 + 8x - 2$

37. $f(x) = \frac{1}{3}x^3 + 2x^2 + 3x - 5$

38. $f(x) = 5 - 8x^3 + 3x^4$

39. Use the graph of $f'(x)$ to the right. Given that $f(0) = 5$, find $f(10)$.



40. Find the area of $y = x^2 - 6x + 2$ between x=4 and x=7.

41. Evaluate $\int_0^3 [(\ln x)(e^{2x})]dx$ correct to 4 decimal places.

42. Given the supply equation for a particular product is given by $p - 0.03x - 20 = 0$ and the demand equation is given by $p + 0.02x - 45 = 0$, find the equilibrium point for the market.

43. Using the equations from the previous question, find the consumer's surplus.

44. Find all absolute extrema for $g(x) = 3x^4 - 16x^3 + 3$ on the interval [-1,2]

45. Find the revenue function for a bottle-cap manufacturer if the marginal revenue is given by $3x(6x^2 + 4)^2$, where x is the number of thousands of golf balls sold.

46. Mark wishes to enclose a rectangular plot of area 216 square yards with fencing. One side of the fence is to be made of special material that costs \$10/yard. The other three sides will be made with fencing that costs \$5/yard. What are the dimensions that will minimize the cost of fence?

47. Find the following anti-derivatives:
    (a) $\int ((-x+2)e^{x^2-4x})\,dx$ (b) $\int \left(\frac{3}{x^2} - \frac{6}{x^3} + \frac{2}{\sqrt{x}}\right) dx$

48. Julie purchases running shoes for \$110. The shoes depreciate linearly over 3 years and have a scrap value of \$0 at this time. How much are the shoes worth after 10 months?

49. Let $f(x) = \dfrac{4}{x-1}$ and $g(x) = \dfrac{x}{x+2}$. Find $(f \circ g)(x)$ and it's domain.

50. It costs a company \$16 to produce each bucket it makes with fixed costs of \$320. The demand for buckets is given by $p = 30 - 0.025x$, where p is the price in dollars and x is the quantity.

    (a) Find both the cost and revenue functions for the company, as functions of quantity, x.

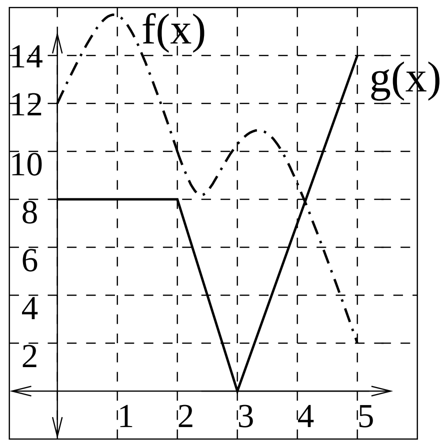
    (b) How many buckets would need to be produced and sold in order to maximuze profit?

    (c) What price should be charged for each bucket in order to maximize profit?

51. How many years will it take your money to triple if it is placed into an account that pays interest at a rate of 3% compounded continuously?

52. Find the distance between the points (2,3,-1) and (-2,4,-3).

Use the graph to the right to answer the following questions:



53. Find $f(g(3))$.

54. Find $f(f(5))$.

55. Find $\int_0^2 g(x)$

56. Find the following anti-derivatives:
(a) $\int \frac{3x^2 - 5x}{\sqrt{x}}\, dx$ (b) $\int \frac{1}{3x \ln(x^4)}\, dx$ (c) $\int \frac{x^2 - 4}{\sqrt{6x^3 - 72x + 10}}\, dx$

57. If $f(x,y) = 2x^3y - e^{x^4 - y} + 5xy^2$, find:

- f_x
- f_y
- f_{xx}
- f_{yy}
- f_{xy}
- f_{yx}

Find all critical points and determine whether each point is a relative minimum, relative maximum, or saddle point.

58. $f(x,y) = -x^2 + 2xy + 2y^2 - 4x - 8y$

59. $f(x,y) = -3x^2 + xy - y^2 - 4x - 3y$

60. $f(x,y) = 3x^2 - 2xy + y^2 + x$

61. $f(x,y) = 2x^3 - 3x^2 - 12x + y^2 - 2y$

62. $f(x, y) = x^2 + xy + 2y^2 - 8x + 3y$

63. BOZO's Bike Company manufactures "Racing Bikes" and "Mountain Bikes". Let x represent the weekly demand for a racing bike and y represent the weekly demand for a mountain bike. The weekly price-demand equations are given by

$$p = 349 - 4x + y, \quad \text{price in dollars for a racing bike}$$

$$q = 446 + 2x - 3y, \quad \text{price in dollars for a mountain bike}$$

How many of each type of bike must BOZO sell in order to maximize the revenue?