Texas A&M University
Mathematics 142, Section 507
Final Exam
December 17, 1997, 8:00 a.m.

Printed name:_____

Signature:_____

Student number:_____

Directions: Show all work for all questions. Don't erase anything unless you are SURE it's wrong.

1. Find $f'(x)$ for each of the following. You need not simplify.

$f(x) = x^2 e^{2x}$

$f(x) = \dfrac{x^2 + 4x}{x - 7}$

$f(x) = \ln(4x - x^3)$

2. Let $f(x, y) = x^2 y + y^2 + xy^4$. Compute $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

3. Compute:
$$\int \frac{x^4}{x^5+5}\,dx$$

4. Compute:
$$\int \frac{e^x + e^{-x}}{e^x}\,dx$$

Given the graph at right, state whether the following are true or false. If I can't read the answer, it's wrong, so you might want to write the whole words "true" and "false".

5. _____ $f(2)$ exists
6. _____ $f'(2)$ exists
7. _____ $\lim_{x\to 2}$ exists
8. _____ f is continuous at 2
9. _____ f has a relative maximum at $x = 2$.

10. _____ If f is continuous, $f'(3)$ doesn't exist, $f'(x) < 0$ for $2 < x < 3$ and $f'(x) > 0$ for $3 < x < 4$ then which of the following must hold?

A. f has a relative maximum at $x = 3$;

B. f has a relative minimum at $x = 3$;

C. f has an inflection point at $x = 3$;

D. f has neither a relative maximum nor a relative minimum at $x = 3$;

E. Cannot conclude any of A,B,C,D.

11. _____ If $f'(3) = 0$ and $f''(3) > 0$ then which of the following must hold?

A. f has a relative maximum at $x = 3$;

B. f has a relative minimum at $x = 3$;

C. f has an inflection point at $x = 3$;

D. Cannot conclude any of A,B,C.

12. _____ Given the graph of $f(x)$, we obtain $f(x - 2)$ by shifting f

A. up;

B. down;

C. to the right;

D. to the left.

13. Consider the following data set and give me the equation you get from a quadratic regression.

x	0	10	14	18	23
y	74	87	96	94	78

14. If $\log_2(2x-6)=5$ then solve for x.

15. If $\frac{\partial f}{\partial x}(1,3)=0$, $\frac{\partial f}{\partial y}(1,3)=0$, $\frac{\partial^2 f}{\partial x^2}(1,3)=-1$ and $\Delta(1,3)=2$, then which of the following must hold?

A. f has a relative maximum at $(1,3)$;

B. f has a relative minimum at $(1,3)$;

C. f has a saddle point at $(1,3)$;

D. Cannot conclude any of A,B,C.

16. _____ Suppose you invest \$5000 in each of 2 bank accounts. The first compounds quarterly at a nominal rate of $r=.05$ and the second compounds monthly at the rate $r=.04$. At the end of a year, which has more money in it?

A. the first;

B. the second;

C. both have the same;

D. can't tell.

17. Suppose $p=10-0.5x$ is the demand and $p=0.5x$ is the supply. Compute the consumer surplus.

18. Find the value of x such that $e^x = 10.5 - 1.4x$.

19. Suppose I have a function f such that $f'(x) = (x-1)^3(2x+1)^2(x+1)$. Find the critical points and state whether f has a relative maximum or minimum at each such critical point.

20. Suppose I have a function $f(x, y)$ and compute $f_x(x, y) = 2x - 3y + 6$ and $f_y(x, y) = -3x + y$. Find the critical points of f and state what conclusion you may draw about each such point from the Second Derivative Test.

21. Suppose you have a choice between two phone services, company A and company B. Company A charges $10 in fixed costs per month and charges $0.15 per minute on long distance calls at night. Company B charges $20 per month in fixed costs and $.05 per minute on night long distance calls. (So if you don't make any long distance calls, clearly A is a better deal. But if you make tons of long distance calls, B is a better deal.) Assuming you make all your long distance calls at night, how many minutes would you have to talk long distance per month in order for B to be cheaper?