MATH 150
Sample Exam 3
Answer Key

Created Summer 2003
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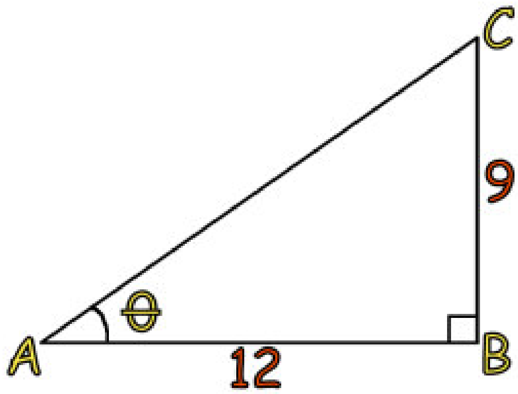
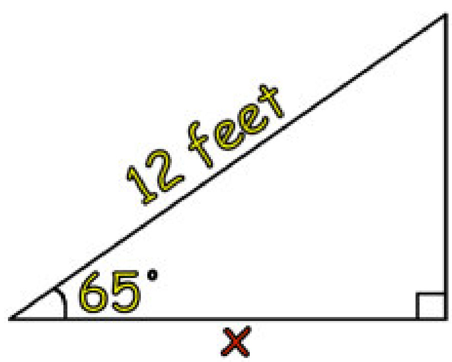
1. Given a right triangle ΔABC, with a 90° angle at point B and with sides BA of length 12 and BC of length 9, if θ = the angle at point A then what is sin(θ) ?



$(\text{Hypotenuse})^2 = 12^2 + 9^2 = 225 \rightarrow$ hyp = 15

So sin(θ) = opp / hyp = 9 / 15 = **3 / 5**

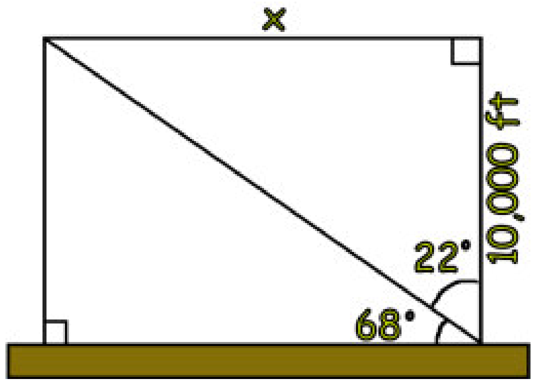
2. If a ladder is placed with an angle of elevation of 65° against a vertical wall and the ladder is 12 feet long then how far is the base of the ladder from the base of the wall? (express your answer to the nearest tenth of a foot).



c = 12 feet, θ = 65°, we are trying to find x.

cos(θ) = adj / hyp → cos(65°) = x / 12 → 12*cos(65°) = x → x ≅ **5.1 ft**

3. A small plane is flying at a constant altitude of 10000 feet. It passes over an observer on the ground. Ten seconds later the observer sees the plane at an angle of elevation of 68°. Find the speed of the plane to the nearest **mile per hour** (recall 1 mile = 5280 feet).



So in 10 seconds the plane traveled a distance of x feet.
$\tan(22°) = x / 10000 \rightarrow 10000*\tan(22°) = x \rightarrow x \cong 4040$ ft

So the plane was traveling at 4040 ft/ 10 sec = 404 ft/sec. Convert that to mph:

$$\frac{404\,feet}{\sec} * \frac{3600\sec}{hour} * \frac{mile}{5280\,feet} \cong$$ **275 miles per hour**.

4. Find the amplitude, period and phase shift of: $y = 3*\cos(\pi*(x + ½))$

Recall given the form: $y = a*\cos(k*(x - b))$,

amplitude = a
period = $2\pi/k$
phase shift = b

So for this problem:

amplitude = 3
period = $2\pi / \pi = 2$
phase shift = -½

5. Find the exact value (no decimals) of cot(5π / 6) + cos(30°)

Recall that 30° = π/6 and cos(30°) = $\frac{\sqrt{3}}{2}$ and sin(30°) = ½.

cot(5π / 6) = cos(5π /6) / sin(5π/6), now use reference angles to simplify

= cos(π/6) / - sin(π/6)

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)}{-\left(\frac{1}{2}\right)}$$

$$= \left(\frac{\sqrt{3}}{2}\right) * (-2) = -\sqrt{3}$$

Thus cot(5π / 6) + cos(30°) = $-\sqrt{3} + \frac{\sqrt{3}}{2} = \boxed{-\frac{\sqrt{3}}{2}}$

6. Verify that: $\tan(x) - \tan(y) = \dfrac{\sin(x-y)}{\cos(x) * \cos(y)}$

Notice that the cos(x)*cos(y) in the denominator of the RHS should be a clue:

LHS = $\tan(x) - \tan(y)$ $= \dfrac{\sin(x)}{\cos(x)} - \dfrac{\sin(y)}{\cos(y)}$

$$= \frac{\sin(x) * \cos(y) - \sin(y)\cos(x)}{\cos(x) * \cos(y)}$$

$$= \frac{\sin(x-y)}{\cos(x) * \cos(y)} = \text{RHS}$$

7. If $\sin(\theta) = -3/5$ and θ is in quadrant III, find $\tan(\theta)$.

Notice in quadrant III, sin() is negative and cos() is negative, thus tan() is positive. Recall also the hypotenuse length is always considered positive.

$\sin(\theta) = opp/hyp = -3/5 \rightarrow opp = -3$ and $hyp = 5$

The Pythagorean Theorem on right triangles says:

$(opp^2 + adj^2) = hyp^2 \quad \rightarrow (-3)^2 + adj^2 = 5^2$

$\rightarrow 9 + adj^2 = 25$

$\rightarrow adj^2 = 16$

$\rightarrow adj = 4$ or -4

So $\tan(\theta) = opp/adj = -3 / 4$ or $-3 / -4$,

as tan() in quad III is positive we conclude: $\tan(\theta) = -3 /-4 =$ **3/4**

8. Write $\sin(3x)*\cos(4x)$ as a sum of trigonometric functions.

Recall $\sin(u)*\cos(v) = ½*[\sin(u + v) + \sin(u – v)]$

Thus

$$\sin(3x)*\cos(4x) = ½*[\sin(3x + 4x) + \sin(3x – 4x)]$$

$$= \mathbf{½*[\sin(7x) + \sin(-x)]}$$