Section 1.6
Solutions and Hints

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for the book:

Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

14. If Ben invests $4000 at 4% interest per year, how much additional money should he invest at 5.5% annual interest to ensure that the interest he receives each year is 4.5% of the total amount invested?

amount invested at 4% = 4000
amount invested at 5.5% = x
total amount invested = 4000 + x

$$4000 * (0.04) + x * (0.055) = (4000 + x) * (0.045)$$
$$160 + 0.055x = 180 + 0.045x$$
$$0.01x = 20$$
$$x = 2000$$

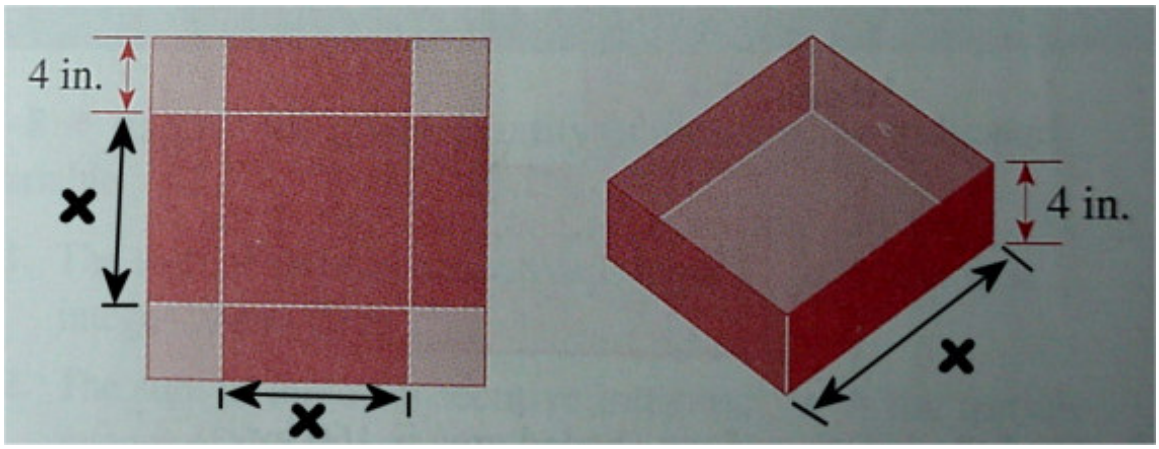
He should invest $2000 at 5.5% to achieve the desired result.

28. A pasture is twice as long as it is wide. Its area is 115,200 sq. ft. How wide is the pasture?

width = W
length = 2W

Area = length * width = 2W * W = 115200
$3W^2 = 115200$
$W^2 = 38400$
W ~= 195.96 ft

36. A box with a square base and no top is to be made from a square piece of cardboard by cutting 4 inch squares from each corner and folding up the sides. The box is to hold 100 cubic inches. How big a piece of cardboard is needed?



Notice the x's have been added, to illustrate the use of the below equations:

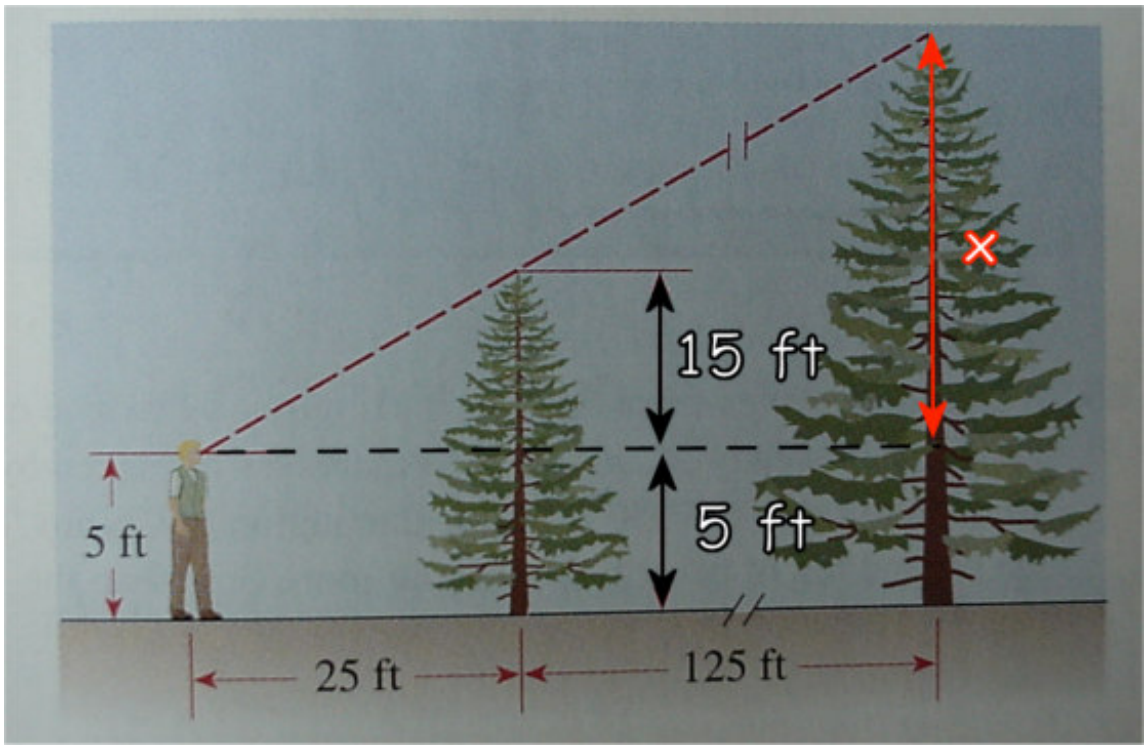
Volume = side*side*height = $x * x * 4 = 100$

$$4x^2 = 100$$
$$x^2 = 25$$
$$x = 5$$

So the original size of the cardboard must be $4 + x = 4 + 5 = 9$ inches per side.

Thus the answer is: **The cardboard sheet must be 9x9 inches.**

48. Bob determines the height of a tall tree by first measuring a smaller one 125 feet away, then moving so his eyes are in the line of sight along the tops of the trees, and measuring how far he is standing from the small tree. Suppose the small tree is 20 ft tall, the man is 25 ft from the small tree and his eye level is 5 ft above the ground.
How tall is the taller tree?



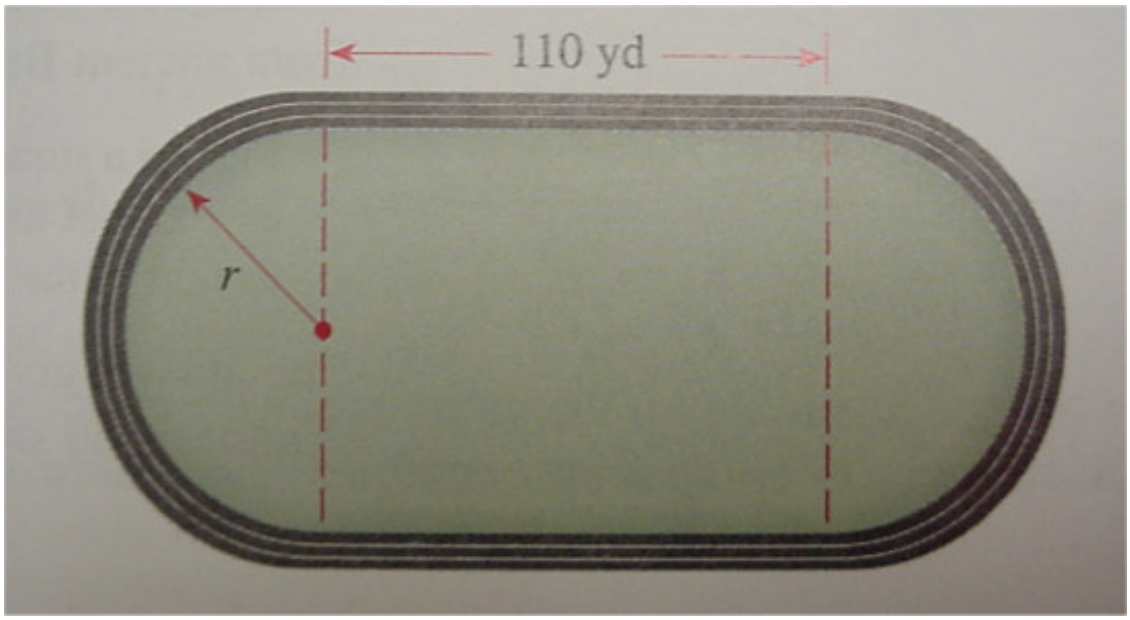
Notice we have two similar triangles:
A small triangle from the man's eyes to the first tree and
a larger one from the man's eyes to the taller tree.

The small triangle has sides of length 25 and 15 feet.
The large triangle has sides length 150 and x.
This would make the height of the taller tree to be x + 5.
All we need to do is determine the value of x.
We will use the properties of similar triangles to do this.

25 / 15 = 150 / x (cross multiply)
25x = 150*15
25x = 2250
x = 90

So **the height of the taller tree is 90+5 = 95 feet.**

75. A running track has the shape shown below with straight sides and semicircle ends. If the length of the track is 440 yards and the two straight sides are each 110 yards long, what is the radius of the semicircular parts (to the nearest yard) ?

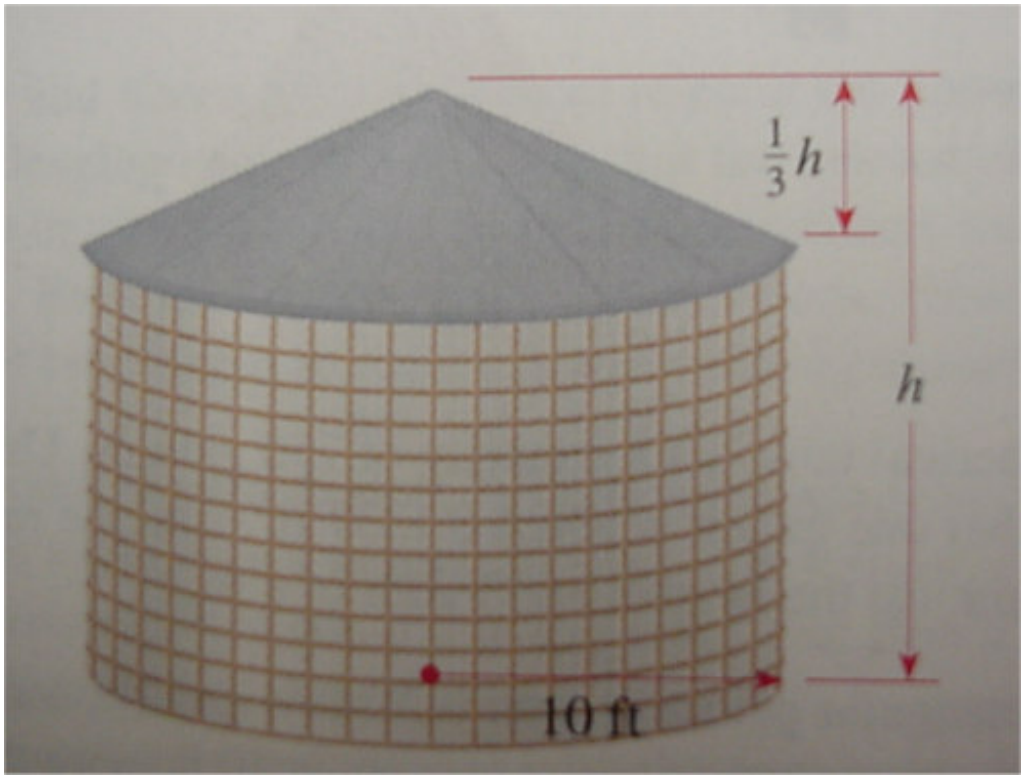


This problem requires no tricks. Notice the two semicircles would form a complete circle if put together. So their combined distance is just the circumference (the distance around) the circle. Recall the circumference = π*diameter = π * 2 * radius. Thus we have the following equation:

440 = 110 + 110 + π * 2r
220 = π*2r
110 / π = r
r ~= 110 / 3.1415 = 35.014 yards

To the nearest yard the radius = 35 yards.

76. A storage bin for corn consists of a cylindrical section of wire mesh, surmounted by a conical tin roof. The height of the roof is one third the entire height of the structure. If the total volume of the structure is 1400π cubic feet and its radius is 10 feet, what is the total height of the structure ?



Recall the volume of a cylinder = $\pi * r^2 *$ (height of cylinder)
Recall the volume of a cone = $(1/3)*\pi * r^2 *$ (height of cone)

In this case we are given r = 10 ft.
The total height = h.
(height of cylinder) = (2/3)*h
(height of cone) = (1/3)*h

Total Volume = 1400π = (volume cylinder) + (volume cone)

$1400\pi = (\pi*10^2*[(2/3) * h]) + ((1/3)*\pi*10^2*[(1/3)*h])$

$1400\pi = (100/3)*\pi*h + (100/9)*\pi*h$

$1400\pi = (400/9)*\pi*h$

$31.5 = h$

So **the total height of the structure is 31.5 feet.**