Section 1.7
Solutions and Hints

by Brent M. Dingle

for the book:

Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

48. Solve for x: [3 / (x – 1)] – [4/ x] ≥ 1

First notice that $x \neq 1$ and $x \neq 0$.
It is also EXTREMELY important to SUBRACT THE ONE – do not try to multiply stuff across the inequality – bad things happen (you would likely end with the result $x \in [-2, 2]$ – which is wrong).

$$\frac{3}{x-1} - \frac{4}{x} - 1 \geq 0$$

$$\rightarrow \frac{3 * x - 4 * (x-1) - (x-1) * x}{(x-1) * x} \geq 0$$

$$\rightarrow \frac{3x - 4x + 4 - x^2 + x}{(x-1) * x} \geq 0$$

$$\rightarrow \frac{4 - x^2}{(x-1) * x} \geq 0$$

And now some thinking is required.
First determine "where" interesting things might happen.
Obviously at x =1 and x = 0 things might be odd (from the denominator)
And at $x^2 = 4$, or rather at x = 2 and x = -2, odd things might happen (from numerator)
Now we check the signs of things on the intervals: (-∞, -2), (-2, 0), (0, 1) , (1, 2), (2, ∞)

	(-∞, -2)	(-2, 0)	(0, 1)	(1, 2)	(2, ∞)
Sign of 4- x^2	-	+	+	+	-
Sign of x-1	-	-	-	+	+
Sign of x	-	-	+	+	+

And we need the final sign to be positive. So (-2, 0) and (1, 2) are the only intervals that will work. Note at x = -2 and x =2 the equation's value is zero, so include those points.
Thus the final answer is $\mathbf{x \in [-2, 0) \cup (1, 2]}$

Graphing it on the number line is left to you.

70. The gas mileage *g* (measured in miles per gallon) for a particular vehicle, driven at *v* miles per hour is given by the formula: $g = 10 + 0.9v - 0.01v^2$, as long as *v* is between 10 mi/hr and 75 mi/hr. For what range of speeds is the vehicle's mileage 30 miles per gallon or better (i.e. $g \geq 30$)

Start with g	≥ 30	and sub in the equation using v:
$10 + 0.9\text{ v} - 0.01v^2$	≥ 30	solve for v
$-20 + 0.9v - 0.01\text{ }v^2$	≥ 0	multiply by 100 to get rid of decimals
$-2000 + 90v - v^2$	≥ 0	divide by –1 -> notice the inequality changes
$2000 - 90v + v^2$	≤ 0	

Now factor $v^2 - 90v + 2000 = (x - 50)(x - 40)$

And we create the signs table, wanting a negative sign when multiplied together (≤ 0)

	[10, 40)	40	(40, 50)	50	(50, 75]
Sign of x – 40	-	0	+	+	+
Sign of x – 50	-	-	-	0	+

So the values that result in something equal 0 are 40 and 50
and the intervals resulting in something < 0 are (40, 50)

Thus the range of speeds is [40, 50]

Thus we conclude: **the vehicle gets 30 miles per gallon or better gas mileage at speeds from 40 mph to 50 mph (inclusive).**

79. A gardener has a 120 feet of deer resistant fence. She wants to enclose a rectangular vegetable garden in her backyard and she wants the enclosed area o be at least 800 sq. feet. What range of values is possible for the length of her garden?

Let the length of the garden = L and the width = W.

The perimeter is 120 feet so: $2L + 2W = 120$
or rather $W = (120 - 2L) / 2 = 60 - L$

The area of the garden would be L*W which must be ≥ 800
So $L*W \geq 800$ and substitute $(60 - L)$ in for W

$L*(60 - L) \geq 800$

$60L - L^2 - 800 \geq 0$ (divide by –1 and change inequality)

$L^2 - 60L + 800 \leq 0$ (now factor)

$(L - 40)(L - 20) \leq 0$

So our = 0 values are 20 and 40. And we make our sign table wanting the result to be < 0

	$(-\infty, 20)$	20	(20, 40)	40	$(40, \infty)$
sign of (L – 20)	-	0	+	+	+
sign of (L – 40)	-	-	-	0	+

So the only interval producing $(L - 40)(L - 20) < 0$ would be (20, 40) and we see that at 20 and 40 the value would = 0. Thus we conclude:

The interval range for the length of her garden would be [20 ft, 40 ft].