Section 1.8
Solutions and Hints

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for the book:

Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

31. Show that the points A(-1, 3), B(3, 11) and C(5, 15) are collinear by showing that d(A, B) + d(B, C) = d(A, C)

$$d(A, B) = \sqrt{(3-(-1))^2 + (11-3)^2} = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5} \approx 8.94427$$

$$d(B, C) = \sqrt{(5-3)^2 + (15-11)^2} = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5} \approx 4.47213$$

$$d(A, C) = \sqrt{(5-(-1))^2 + (15-3)^2} = \sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5} \approx 13.4164$$

And we see that $4\sqrt{5} + 2\sqrt{5} = 6\sqrt{5}$ and thus we have shown that
d(A, B) + d(B, C) = d(A, C) and thus the points must be collinear.

82. Show that $x^2 + y^2 - 4x + 10y + 13 = 0$ represents a circle and find the circle's center and radius.

To show the equation represents a circle you must demonstrate that it can be put in the form of: $(x - h)^2 + (y - k)^2 = r^2$, notice by doing this you will also answer the rest of the question. Completing the square is usually the method of choice for this type of problem.

$x^2 + y^2 - 4x + 10y + 13 \rightarrow (x^2 - 4x + \_\_\_) + (y^2 + 10y + \_\_\_) = -13$
$\rightarrow (x^2 - 4x + (-4/2)^2) + (y^2 + 10y + (10/2)^2) = -13 + (4/2)^2\ (10/2)^2$
$\rightarrow (x - 2)^2 + (y + 5)^2 = 16$
$\rightarrow (x - 2)^2 + (y + 5)^2 = 4^2$

So the center of the circle is (2, -5) with a radius of 4.

89. Find the area of the region that lies outside the circle: $x^2 + y^2 = 4$, but inside the circle: $x^2 + y^2 - 4y - 12 = 0$

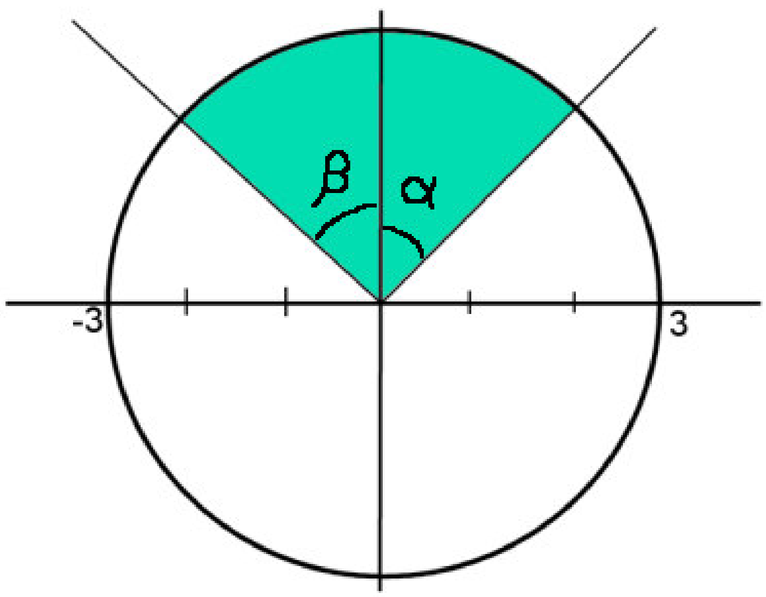
To solve this all you need to do is see that

1. the first circle is centered at (0, 0) with radius 2
2. the second circle can be rewritten as $(x + 0)^2 + (y - 2)^2 = 16$ so it is centered at (0, 2) with radius 4
3. the first circle is completely contained within the second
4. the area of a circle = π * $(\text{radius})^2$

Thus the answer is: (area of circle 2) – (area of circle 1)
$= \pi*16 - \pi*4$
$= 12\pi$

90. Sketch the region in the coordinate plane that satisfies both the inequalities: $x^2 + y^2 \le 9$ and $y \ge | x |$. What is the area of this region.

When you sketch this you should discover you have a circle of radius 3 centered at (0,0) and a V with its point at (0,0) that goes through (-1, 1) and (1, 1). Specifically you get:



where you need to determine the area of the shaded region. There are several ways to go from here. The easiest is to see that the angle of the V is 90° (each line is at a 45° angle from the y-axis by tan $\alpha = 1/1$ and tan $\beta = 1/-1$) which means it is cutting out 90/360 or ¼ of the circle. Thus the area is (¼)*(area of circle) = $(¼)*\pi*3^2$ = **$(9/4)\pi$**.