Section 2.3
Solutions and Hints

by Brent M. Dingle

for the book:
<u>**Precalculus, Mathematics for Calculus 4th Edition**</u>
by James Stewart, Lothar Redlin and Saleem Watson.

13. Express the following as a formula. Use the given information to find the constant of proportionality:
y is directly proportional to x. If x = 6 then y = 42

$y = k*x$
$42 = k * 6 \rightarrow k = 7$ Which means $\mathbf{y = 7x}$

22. Express the following as a formula. Use the given information to find the constant of proportionality:
M is jointly proportional to a, b and c and inversely proportional to d.
If a and d have the same value and if b and c are both 2 then M = 128.

$M = k*(a*b*c) / d$
If a = d we can sub a in for d and they will cancel:
$M = k*(a*b*c) / a$
$\rightarrow M = k*(b*c)$
Now put in 2 for b and c and 128 for M and solve for k:
$128 = k*(2*2) \rightarrow 128 = 4k \rightarrow k = 32$
And we see that:

$$\mathbf{M = 32*(a*b*c) / d}$$

24. The period of a pendulum (the time elapsed during one complete swing of the pendulum) varies directly to with the square root of the length of the pendulum.

24 a. Express this relationship by writing an equation.

Let T = the period of the pendulum.
Let L = the length of the pendulum.

$T = k * \sqrt{L}$

24 b. In order to double the period, how would we have to change the length, L, of the pendulum?

So we want to go from T to $2T$, and we will assume k stays constant.
For example say our original period, T_{orig}, was 1, then our new period, T_{new} would be 2.

Let L_{orig} = the original length of the pendulum.
Let L_{new} = the required length of the pendulum to double our period.

So $\underline{T_{orig}} = k * \sqrt{L_{orig}}$

And $T_{new} = 2 * \underline{T_{orig}} = 2 * k * \sqrt{L_{orig}} = k * \sqrt{L_{new}}$

So now we can solve for L_{new} in terms of L_{orig}.

$2 * k * \sqrt{L_{orig}} = k * \sqrt{L_{new}}$ → $4 * k^2 * L_{orig} = k^2 * L_{new}$ *(square both sides)*

→ $4 * L_{orig} = L_{new}$ *(divide out k^2)*

And we have our answer.

To double the period we must multiply the pendulum length by 4.

32. The heat experienced by a hiker at a campfire is proportional to the amount of wood on the fire and inversely proportional to the cube of her distance from the fire. If she is 20 feet from the fire and another person doubles the amount of wood burning then how far from the fire would she need to be to feel the same heat as before?

Let h = quantity of heat felt
Let d = distance from the fire
Let w = quantity of wood burning on the fire
Let k = some constant of proportionality

$h = k*w / d^3$

Let $h_{orig} = k*w / (d_{orig})^3$,   notice we are given $d_{orig} = 20$ feet
Let $h_{new} = k*(2w) / d^3$,   notice we will need to find d

We want $h_{orig} = h_{new}$ and we will need to find distance, d, that makes that true. So

$h_{orig} = h_{new}$ → $k*w / 20^3 = k*2w / d^3$ *multiply d^3 to the left side*
→ $d^3 * k*w / 20^3 = k*2w$ *multiply 20^3 to the right side*
→ $d^3 * k*w = k*2w*20^3$ *divide both sides by k*w*
→ $d^3 = 2*20^3$
→ $d = \sqrt[3]{2*20^3} = 20*\sqrt[3]{2}$ ~= **25.198421 feet**