

Section 2.4

Solutions and Hints

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for the book:

Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

This section is all about average rate of change. While the book makes it seem complicated it is not. Here it is easy:

1. You have some function $f(x)$.
2. You pick two x values say x_1 and x_2 .
3. You then put those two values into $f(x)$ and get $f(x_1) = y_1$ and $f(x_2) = y_2$.
4. You then calculate the slope of the line from (x_2, y_2) to (x_1, y_1) .
5. This slope is the average rate of change of $f(x)$ from x_1 to x_2 .

- 20. A man is running around a circular track 200 meters in circumference. An observer uses a stopwatch to time each lap, obtaining the following data:**

Time in seconds	Distance in meters
32	200
68	400
108	600
152	800
203	1000
263	1200
335	1400
412	1600

- 20 a. What was the man's average speed (rate) between 68 s and 152 s?**

$$\begin{aligned} \text{So } x_1 &= 68 \rightarrow y_1 = 400 \\ x_2 &= 152 \rightarrow y_2 = 800 \end{aligned}$$

$$\text{avg rate} = (y_2 - y_1) / (x_2 - x_1) = (800 - 400) / (152 - 68) = 100/21 \approx \mathbf{4.7619 \text{ m/s}}$$

20 b. What was the man's average speed between 263 s and 412 s?

$$\text{So } x_1 = 263 \rightarrow y_1 = 1200$$

$$x_2 = 412 \rightarrow y_2 = 1600$$

$$\begin{aligned} \text{avg rate} &= (y_2 - y_1) / (x_2 - x_1) \\ &= (1600 - 1200) / (412 - 263) \\ &= 400/19 \approx \mathbf{2.6846 \text{ m/s}} \end{aligned}$$

**20 c. Calculate the man's speed for each lap.
Is he slowing down, speeding up or neither?**

Time in seconds	Distance in meters	Average Speed (rate) in m/s
32	200	$(200 - 0)/(32 - 0) = 25/4 \approx \mathbf{6.25}$
68	400	$(400 - 200)/(68 - 32) = 50/9 \approx \mathbf{5.56}$
108	600	$(600 - 400)/(108 - 68) = \mathbf{5}$
152	800	$(800 - 600)/(152 - 108) = 50/11 \approx \mathbf{4.55}$
203	1000	$200/(203 - 152) = 200/11 \approx \mathbf{3.92}$
263	1200	$200 / (263 - 203) = 10/3 \approx \mathbf{3.33}$
335	1400	$200 / (335 - 263) = 25/9 \approx \mathbf{2.78}$
412	1600	$200 / (412 - 335) = 200/77 \approx \mathbf{2.60}$

Every lap the man's average speed is decreasing thus **he is slowing down every lap.**