

Section 2.6

Solutions and Hints

by Brent M. Dingle

for the book:

Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

You MUST be able to express a quadratic function: $f(x) = ax^2 + bx + c$ in standard form:

$$f(x) = a(x - h)^2 + k.$$

where $h = -b / (2a)$
 $k = c - b^2 / (4a)$

The graph of the function will always be the parabola with vertex (h, k) .

If $a > 0$ then the parabola will open upwards

and $k = c - b^2 / (4a)$ will be the minimum of the function.

If $a < 0$ then the parabola will open downwards

and $k = c - b^2 / (4a)$ will be the maximum of the function.

Notice the book would have you complete the square for every problem. If you are quick at doing that then do that, otherwise memorizing the above equations for h and k will save you a great deal of time.

26. Find the maximum or minimum of $f(x) = 1 + 3x - x^2$

$$a = -1$$

$$b = 3$$

$$c = 1$$

$$h = -b / (2a) = -3 / (2 \cdot -1) = 3/2$$

$$k = c - b^2 / (4a) = 1 - 9 / (4 \cdot -1) = 1 - (-9/4) = 1 + 9/4 = 13/4$$

Since $a < 0$ the parabola opens downwards and

a maximum occurs at $x = 3/2$, $y = 13/4$.

40. A ball is thrown across a playing field. Its path is given by the equation $y = -0.005x^2 + x + 5$, where x is the horizontal distance traveled by the ball and y is the height of the ball above the ground (measured in feet).

40 a. What is the maximum height of the ball along this path?

$$a = -0.005$$

$$b = 1$$

$$c = 5$$

$$h = -b / (2a) = -1 / (2 * -0.005) = 100$$

$$k = c - b^2 / (4a) = 5 - 1 / (4 * -0.005) = 55$$

Since $a < 0$ the ball attains a maximum at $x = 100$ and $y = 55$.

Thus **the maximum height is 55 feet.**

40 b. How far has the ball traveled horizontally when it hits the ground?

Notice it hits the ground when $y = 0$.

From part (a) we have the standard form of the equation to be:

$$y = -0.005(x - 100)^2 + 55$$

Set that equal to zero and solve for x :

$$0 = -0.005(x - 100)^2 + 55, \text{ divide everything by } -0.005$$

$$0 = (x - 100)^2 - 11000$$

$$0 = x^2 - 200x + 10000 - 11000$$

$$0 = x^2 - 200x - 1000 \quad \text{Solve via quadratic equation}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{200 \pm \sqrt{40000 - (-4000)}}{2} = \frac{200 \pm \sqrt{44000}}{2}$$

$x \approx 204.88$ feet (or -4.88 feet which is not a valid result as it is negative)