Section 2.7
Solutions and Hints

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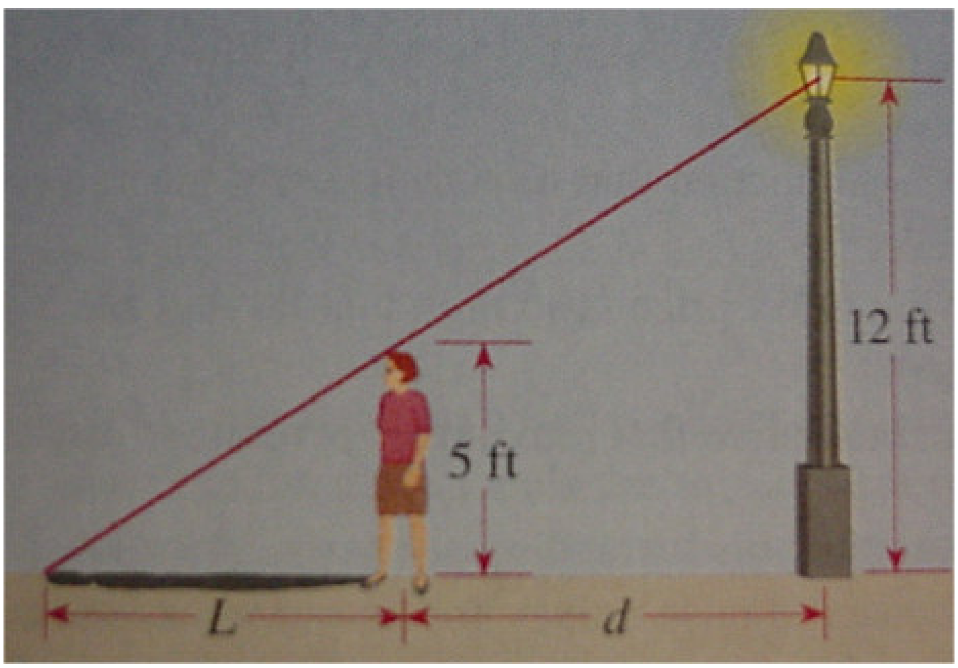
for the book:
<u>Precalculus, Mathematics for Calculus 4th Edition</u>
by James Stewart, Lothar Redlin and Saleem Watson.

12. A woman 5 feet tall is standing near a street lamp that is 12 feet tall. Find a function that models the length of her shadow, L, in terms of her distance, d, from the base of the lamp.



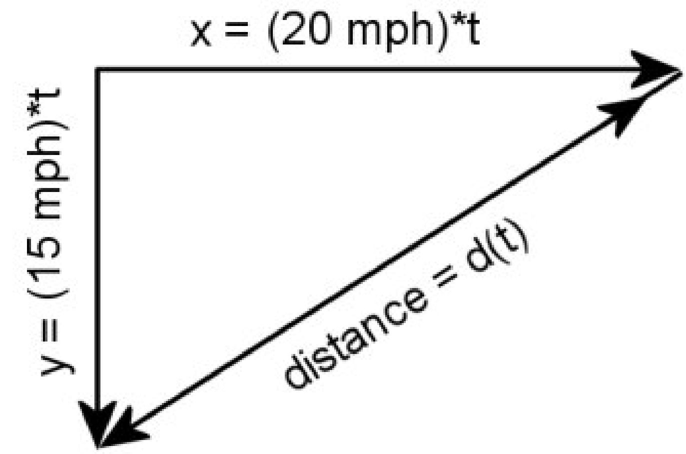
Use similar triangles:

$$\frac{L}{5} = \frac{L+d}{12} \to 12L = 5L + 5d \to 12L - 5L = 5d \to 7L = 5d \to L = \frac{5}{7}d$$

So L as a function of d is **L(d) = (5/7)*d**

13. Two ships leave port at the same time. One sails south at 15 mph and the other sails east at 20 mph. Find a function that models the distance, d between the ships in terms of time t (in hours) elapsed since their departure.

Consider the picture:



So $(\text{distance})^2 = [d(t)]^2 = x^2 + y^2 = (15t)^2 + (20t)^2 = 225t^2 + 400t^2 = 625t^2$

Now we take the square root of both sides (keeping only the positive result):

$d(t) = 25t$, with $t \geq 0$

20. Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

Let x and y be our two numbers.
We know: $x + y = 100$ and we want to minimize $x^2 + y^2$.

So from $x + y = 100$ we get $y = 100 – x$.
We will put that into the second equation to get $f(x) = x^2 + (100 – x)^2$.

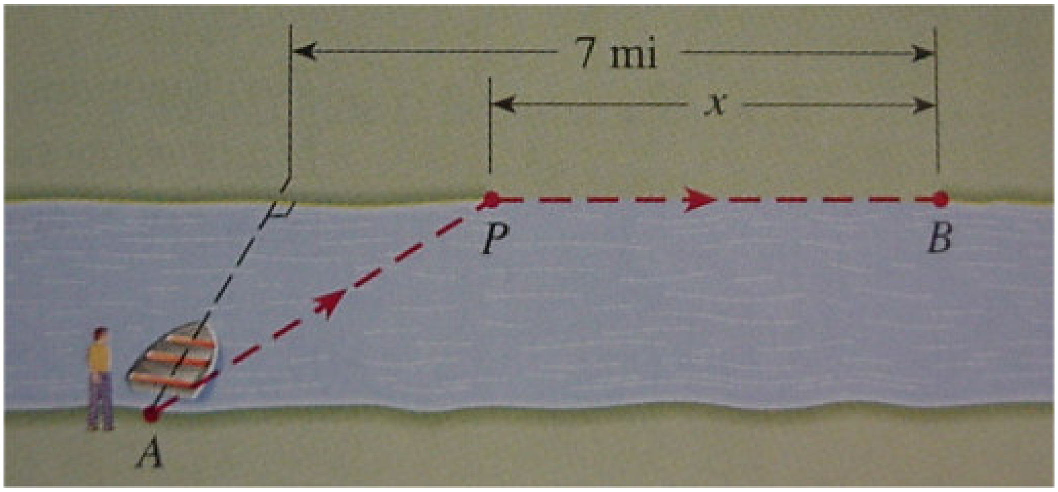
Now we want to minimize:

$$f(x) = x^2 + 10000 – 200x + x^2 = 2x^2 – 200x + 10000.$$

You can do this by graphing the equation on your calculator and estimating, or you can use what you learned in section 2.6:
$a = 2$, $b = -200$, $c = 10000$ and $h = -b/(2a)$, $k = c – b^2/(4a)$
$h = -(-200/2*2) = 50$ and $k = 10000 – 40000/8 = 9900$

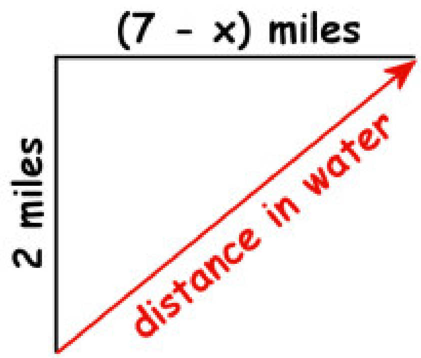
So at $x = h = 50$ we minimize $f(x)$. Putting 50 back into $x + y = 100$ we discover $y = 50$.
Thus the answer is **$x = 50$ and $y = 50$**.

34. A man stands at a point A on the bank of a straight river, 2 miles wide. To reach point B, 7 miles downstream on the opposite bank, he first rows his boat to point P on the opposite bank and then walks the remaining distance x to point B. He can row at a speed of 2 mph and can walk at a speed of 5 mph. (Ignore river current).



34 a. Find a function that models the time needed for the trip.

So the distance in the water is calculated by looking at the triangle:



d_{water} $= 2^2 + (7 - x)^2 = 4 + 49 - 14x + x^2 = x^2 - 14x + 53.$

d_{land} $= x$

time $= d_{water} * 2\text{mph} + d_{land} * 5\text{mph} = (x^2 - 14x + 53)*2 + x*5$

$= 2x^2 - 28x + 106 + 5x$

time $= 2x^2 - 23x + 106$ hours

34 b. Where should he land so that he reaches B as soon as possible?

Again you may just enter the function found in part (a) into your calculator, graph it and guess where the minimum occurs or you can actually find the solution by applying what you learned in section 1.6: $a = 2$, $b = -23$, $c = 106$ and $h = -b/(2a)$, $k = c - b^2/(4a)$

$h = 23 / 4$ and $k = 106 - 529/8 = 39.875$

So time is at a minimum when $x = h = 23/4$ miles.

Thus he should land 23/4 = 5.75 miles upstream of point B.