Section 2.9
Solutions and Hints

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for the book:
Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

Memorize either or both of these:

A function is one to one if $x1 \neq x2$ then $f(x1) \neq f(x2)$.

A function is one to one if $f(x1) = f(x2)$ then $x1 = x2$.

Memorize the 3 steps at the bottom of page 233 for finding the inverse of a one-to-one function. You will be expected to be able to do this:

1. Write $y = f(x)$
2. If possible, solve this equation for x in terms of y (e.g. $x = 3*y - 5$).
3. Interchange x and y (e.g. $y = 3*x - 5$). The resulting equation is $y = f^{-1}(x)$.

18 a. If f(5) = 18, find f^{-1}(18)

Here is a neat trick, that will help later.

Recall $f(f^{-1}(x)) = x = f^{-1}(f(x))$

Given: $f(5) = 18$ So take $f^{-1}(\ )$ of both sides

       $f^{-1}(f(5)) = f^{-1}(18)$ Simplify

       $5 = f^{-1}(18)$

18 b. If f^{-1}(4) = 2, find f(2)

Use the same trick.

Given: $f^{-1}(4) = 2$ Take $f(\ )$ of both side

       $f(f^{-1}(4)) = f(2)$ Simplify

       $4 = f(2)$

23. Show f and g are inverses of each other.
   $f(x) = 2x - 5, \qquad g(x) = (x + 5) / 2$

Show that $f \circ g = f(g(x)) = x$ and then show $g \circ f = g(f(x)) = x$

$f \circ g = f(g(x)) = f((x + 5)/2) = 2*((x + 5)/2) - 5 = (x + 5) - 5 = x$

$g \circ f = g(f(x)) = g((2x - 5)) = ((2x - 5) + 5) / 2 = (2x) / 2 = x$

So $f \circ g = g \circ f = x$ and thus by the Property of Inverse Functions f and g are inverses.

32. Find the inverse of f(x) = 6 – x.

$y = 6 - x$ Let $y = f(x)$, then solve for x
$x + y = 6$
$x = 6 - y$ Interchange the x and the y
$y = 6 - x$ And now $y = f^{-1}(x)$

$f^{-1}(x) = 6 - x.$

38. Find the inverse of f(x) = (x – 2) / (x + 2)

$y = (x - 2) / (x + 2)$ Let $y = f(x)$ and then solve for x
                              You will need to actually remember "synthetic division"
                              You get 1 with a remainder of negative 4

$y = 1 - 4 / (x + 2)$
$y - 1 = -4 / (x + 2)$
$(x + 2)*(y - 1) = -4$
$x + 2 = -4 / (y - 1)$
$x = -4 / (y - 1) - 2 = -2 - 4/(y - 1) = -2 - \dfrac{4}{y-1}$ Interchange the x and y

$y = -2 - \dfrac{4}{x-1}$

$f^{-1}(x) = -2 - 4/(x -1)$