Section 3.3
Solutions and Hints

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for the book:
<u>Precalculus, Mathematics for Calculus 4th Edition</u>
by James Stewart, Lothar Redlin and Saleem Watson.

40. Find all the real zeros of $P(x) = 3x^3 - 5x^2 - 8x - 2$.

Factors of –2: -2, -1, 1, 2   Factors of 3: 1, 3

Possible RATIONAL roots: -2, -2/3, -1, -1/3, 1, 1/3, 2, 2/3

So pick one and divide it out to see what happens.
If it divides evenly things are good. If not, pick another.
For brevity we will begin with –1/3. Or rather we will divide by (x + 1/3)

First we bring down the leading 3:

-1/3	3	-5	-8	-2
	3			

Multiply –1/3 * 3 and place the result under the –5 and then add.

-1/3	3	-5	-8	-2
		-1		
	3	-6		

Multiply –1/3 * -6 and place the result under the –8 and then add.

-1/3	3	-5	-8	-2
		-1	2	
	3	-6	-6	

Multiply –1/3 * -6 and place the result under the –2 and then add.

-1/3	3	-5	-8	-2
		-1	2	2
	3	-6	-6	0

40. continued from previous page…

Thus $(3x^3 – 5x^2 – 8x – 2) / (x + 1/3) = 3x^2 – 6x – 6$.

So we need to now solve $3x^2 – 6x – 6 = 0$, or just $x^2 – 2x – 2 = 0$. And we see that by the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4*1*(-2)}}{2*1} = \frac{2 \pm \sqrt{12}}{2}$

Thus x ~= 2.732 or –0.732

Or rather the zeros of $(3x^3 – 5x^2 – 8x – 2)$ occur at **x = -1/3 or 2.732 or –0.732**

Notice it would be better to leave the roots expressed as irrational numbers, the decimals are inaccurate.

62. Find integers that are upper and lower bounds for the real zeros of the polynomial, $P(x) = 2x^3 – 3x^2 – 8x – 12$.

Factors of –12:	-12, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 12
Factors of 2:	1, 2
Possible rational zeros:	-12, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 12
	-6, -3, -2, -3/2, -1, -1/2, 1/2, 1, 3/2, 2, 3, 6
Discarding the duplicates and ordering them gives:	-12, -6, -4, -3, -2, -3/2, -1, -1/2, 1/2, 1, 3/2, 2, 3, 4, 6, 12

Obviously we need to perform synthetic division using each value until we find one which has no negative entry in the bottom row (the upper bound) and one which has alternately nonpositive and nonnegative entries (the lower bound).

Notice that –12 and 12 are likely to work – but not necessarily, irrational numbers are also real numbers. And a possible zero might occur at $12 + \sqrt{7}$ which is greater than 12. However it is a good starting point.

For 12 we get the following synthetic division:

12	2	-3	-8	-12
		24	252	2928
	2	21	244	2916

Notice these are all nonnegative thus 12 will work as an upper bound for all real roots.

For -12 we get the following synthetic division:

-12	2	-3	-8	-12
		-24	324	-3792
	2	-27	316	-3804

Notice each entry alternates sign so –12 will work as a lower bound for all real roots.

Thus –12 is a lower bound and 12 is an upper bound.

Notice there may exist 'tighter' bounds such as –6 and 4, for example.