Section 3.5
Solutions and Hints

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for the book:
<u>Precalculus, Mathematics for Calculus 4th Edition</u>
by James Stewart, Lothar Redlin and Saleem Watson.

40. Find all zeros of $x^3 - 7x^2 + 17x - 15$.

Recall to find the zeros of a polynomial the "easiest" method is to factor the polynomial. For example if $x^3 - 7x + 17x - 15 = 0$ can be factored such that $(x - a)(x - b)(x - c) = 0$ then a, b and c are called the zeros of the polynomial.

The factors of -15 are: -15, -5, -3, -1, 1, 3, 5, 15
The factors of 1 are: 1 (1 is the coefficient in front of the highest power, x^3, term)
So the possible rational solutions are: -15, -5, -3, -1, 1, 3, 5, 15

Synthetic division shows that 3 is the only rational solution:

3	1	-7	17	-15
		3	-12	15
	1	-4	5	0

Thus $x^3 - 7x + 17x - 15 = (x - 3)(x^2 - 4x + 5)$

So x = 3 is a zero of $x^3 - 7x + 17x - 15$ and we can use the quadratic formula to find the other 2 (complex) zeros by solving $x^2 - 4x + 5 = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4*1*5}}{2*1} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

Thus the zeros of $x^3 - 7x + 17x - 15$ are x = **3, 2 + i, 2 – i**.

48. Find all zeros of $x^4 - 2x^3 - 2x^2 - 2x - 3$.

This works just like problem 40, first we look for the rational solutions:

Factors of –3: -3, -1, 1, 3
Factors of 1: 1 (1 is the coefficient in front of the highest power, x^4, term)

So the possible rational zeros are: -3, -1, 1, 3

Synthetic division shows that -1 is a rational solution:

-1	1	-2	-2	-2	-3
		-1	3	-1	3
	1	-3	1	-3	0

So $x^4 - 2x^3 - 2x^2 - 3 = (x + 1)(x^3 - 3x^2 + x - 3)$

Notice the possible rational zeros of $x^3 - 3x^2 + x - 3$
are the same as those for $x^4 - 2x^3 - 2x^2 - 3$, this should not be surprising.

Working now on $x^3 - 3x^2 + x - 3$ we find through synthetic division that 3 is the only other rational solution:

3	1	-3	1	-3
		3	0	3
	1	0	1	0

So $x^4 - 2x^3 - 2x^2 - 3 = (x + 1)(x - 3)(x^2 + 1)$

Solving $(x^2 + 1) = 0$ gives $x^2 = -1$ or rather $x = \pm\sqrt{-1} = \pm i$

Thus the zeros of $x^4 - 2x^3 - 2x^2 - 3$ are x = **-1, 3, -*i*, *i***.