Section 3.6
Solutions and Hints

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for the book:
Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

Aside from being able to draw graphs of functions, the important part of this section is gaining the ability to determine the asymptotes of functions.

Vertical asymptotes are probably the easiest. They occur where the function is undefined. Usually this occurs when the denominator portion of a function is zero or when the stuff inside a square root (or even power root) is negative. So you will need to 'solve' for the zeros of the denominator or similar.

Horizontal asymptotes are little tricky, but not too bad. Given a function which has a numerator and denominator (a rational function), three things can happen:

1. if the degree of the numerator is greater than the degree of the denominator then NO horizontal asymptote exits.
2. if the degree of the numerator is less than the degree of the denominator then a horizontal asymptote occurs at $y = 0$.
3. if the degree of the numerator equals the degree of the denominator then there IS a horizontal asymptote.
   Let a = the coefficient in front of the highest powered 'x' in the numerator.
   Let b = the coefficient in front of the highest powered 'x' in the denominator.
   Then the horizontal asymptote occurs at $y = a / b$.

Notice there is no 'solving' for anything to find these.

Slant asymptotes, also called **oblique asymptotes** are the most 'difficult.' These will ONLY occur when the degree of the numerator is 1 greater than the degree of the denominator. To find them you will need to divide stuff out. Specifically:
If $r(x) = P(x) / Q(x)$ where the degree of $P(x) = 1 +$ the degree of $Q(x)$
then we can divide stuff out to get $r(x)$ in the form of $r(x) = ax + b + R(x)/Q(x)$.
(Notice the $R(x)$ will be the remainder of our division)
The slant asymptote is then defined as: $ax + b$.

52. Find the slant asymptote and vertical asymptote the below function, and graph it.

$$r(x) = \frac{3x - x^2}{2x - 2} = \frac{-x^2 + 3x + 0}{2x - 2}$$

<u>Vertical Asymptote</u>:

Set the denominator = 0 and solve for x:

$2x - 2 = 0$

→ $2x = 2$

→ $x = 1$

So there is a **vertical asymptote at x = 1**

<u>Slant Asymptote</u>:

Divide stuff out:

		$-½*x$	$+1$	remainder 2
$2x - 2$	$-x^2$	$+ 3x$	$+ 0$	
-	$-x^2$	x		
	0	$2x$	0	
-		$2x$	-2	
		0	2	

Thus we see $r(x) = \frac{3x - x^2}{2x - 2} = \frac{-x^2 + 3x + 0}{2x - 2} = (1/2)x + 1 + \frac{2}{2x - 2}$

So there is a **slant asymptote at (½)x + 1**

The graphing is left for you to complete.