Section 4.1
Solutions and Hints

by Brent M. Dingle

for the book:
Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

If you haven't memorized the following, you might as well do so now:

Compound Interest

$$A(t) = P * \left(1 + \frac{r}{n}\right)^{n*t}$$

where $A(t)$ = amount after t years.
P = principal (or initial amount).
r = interest rate per year.
n = number of times interest is compounded per year.
t = time in years.

Continuously Compounded Interest

$$A(t) = Pe^{r*t}$$

where $A(t)$ = amount after t years.
P = principal (or initial amount).
r = interest rate per year.
t = time in years.

46. A 50 gallon barrel is filled completely with pure. Salt water with a concentration of 0.3 lb/gal is then pumped into the barrel, and the resulting mixture overflows at the same rate. The amount of salt in the barrel at time *t* is given by:

$$Q(t) = 15*(1 - e^{-0.04 * t})$$

where *t* is measured in minutes and Q(*t*) is measured in pounds.

46 a. How much salt is in the barrel after 5 minutes?

$Q(5) = 15 * (1 - e^{-0.04 * 5})$ ~= **0.297 lbs**.

46 b. How much salt is in the barrel after 10 minutes?

$Q(5) = 15 * (1 - e^{-0.04 * 10})$ ~= **0.588 lbs**.

46 c. Draw the graph of Q(t)

This is left for you to complete.

46 d. Use the graph in part (c) to determine the value that the amount of salt in the barrel approaches as *t* becomes large. Is this what you expect.

You should see that **the amount approaches 15 pounds** as *t* becomes large. This should make sense because the barrel will eventually be completely full of water that has a 0.3 lb/gal concentration of salt in it → 50 gallons * 0.3 pounds / gallon = 15 pounds.

49. If $10 000 is invested at an annual interest rate of 10% per year, compounded semiannually, find the value of the investment after the given number of years:

49 a. 5 years

This will obviously use the formula: $A(t) = P * \left(1 + \frac{r}{n}\right)^{n*t}$

P = 10 000
r = 0.10
n = 2, since semiannually means 2 times per year.
t = 5
A(t) = what we need to find.

$$A(t) = 10000 * \left(1 + \frac{0.1}{2}\right)^{2*5} = 10000 * (1.05)^{10} \approx 16288.95$$

So after 5 years the investment would be worth **$16,288.95**

49 b. 10 years

Everything is the same except $t = 10$

$$A(t) = 10000 * \left(1 + \frac{0.1}{2}\right)^{2*10} = 10000 * (1.05)^{20} \approx 26532.98$$

So after 10 years the investment would be worth **$26,532.98**

49 c. 15 years

Everything is the same except $t = 15$

$$A(t) = 10000 * \left(1 + \frac{0.1}{2}\right)^{2*15} = 10000 * (1.05)^{30} \approx 43219.42$$

So after 15 years the investment would be worth **$43,219.42**

50. If $4000 is borrowed at a rate of 16% interest per year, compounded quarterly, find the amount due at the end of the given number of years.

50 a. 4 years

This will obviously use the formula: $A(t) = P * \left(1 + \frac{r}{n}\right)^{n*t}$

P = 4 000
r = 0.16
n = 4, since quarterly means 4 times per year.
t = 4
A(t) = what we need to find.

$$A(t) = 4000 * \left(1 + \frac{0.16}{4}\right)^{4*4} = 4000 * (1.04)^{16} \approx 7491.92$$

So after 4 years the amount due would be **$7,491.92**

50 b. 6 years

This is the same except $t = 6$

$$A(t) = 4000 * \left(1 + \frac{0.16}{4}\right)^{4*6} = 4000 * (1.04)^{24} \approx 10253.22$$

So after 6 years the amount due would be **$10,253.22**

50 c. 8 years

This is the same except $t = 8$

$$A(t) = 4000 * \left(1 + \frac{0.16}{4}\right)^{4*8} = 4000 * (1.04)^{32} \approx 14032.24$$

So after 8 years the amount due would be **$14,032.24**