Section 4.4
Solutions and Hints

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for the book:

<u>Precalculus, Mathematics for Calculus 4th Edition</u>
by James Stewart, Lothar Redlin and Saleem Watson.

28. Solve: $x^2*10^x - x*10^x = 2*(10^x)$

$x^2*10^x - x*10^x = 2*(10^x)$ Divide both sides by 10^x.
$x^2 - x = 2$
$x^2 - x - 2 = 0$ Now factor
$(x - 2)(x + 1) = 0$

$x = 2$ or $x = -1$

32. Solve: $e^{2x} - e^x - 6 = 0$

$e^{2x} - e^x - 6 = 0$ This is actually example 4 in the book on page 367.
$(e^x)^2 - e^x - 6 = 0$
$(e^x - 3)(e^x + 2) = 0$

Now solve: $(e^x - 3) = 0$ and $(e^x + 2) = 0$

$(e^x - 3) = 0 \rightarrow e^x = 3 \rightarrow \ln(e^x) = \ln(3) \rightarrow x = \ln(3)$

$(e^x + 2) = 0 \rightarrow e^x = -2 \rightarrow \ln(e^x) = \ln(-2) \rightarrow$ no solution

So the only answer is **$x = \ln(3)$**

34. Solve: $e^x - 12e^{-x} - 1 = 0$

This one looks tough, but is not. Remember that the only real trick you have is when you have an e^{2x}, and you change it to $(e^x)^2$ and then you factor. So this problem is missing that e^{2x} term, thus you might think… "If I multiply both sides by e^x what will happen?"

$e^x - 12e^{-x} - 1 = 0$	Multiply both sides by e^x.
$(e^x)^2 - 12 - 1 * e^x = 0$	Rewrite stuff a little better
$(e^x)^2 - e^x - 12 = 0$	Now factor
$(e^x - 4)(e^x + 3) = 0$	

Now solve for: $(e^x - 4) = 0$ and $(e^x + 3) = 0$

$(e^x - 4) = 0 \rightarrow e^x = 4 \rightarrow \ln(e^x) = \ln(4) \rightarrow x * \ln(e) = \ln(4) \rightarrow x = \ln(4)$

$(e^x + 3) = 0 \rightarrow e^x = -3 \rightarrow x = \ln(-3) \rightarrow$ no solution

So the only answer is **$x = \ln(4)$**

46. Solve for x: $\log_5 x + \log_5(x + 1) = \log_5 20$

$\log_5(x) + \log_5(x+1) = \log_5(20)$	Use log properties
$\rightarrow \log_5(x * (x + 1)) = \log_5(20)$	
$\rightarrow \log_5(x^2 + x) = \log_5(20)$	Now use prop: if a = b then $5^a = 5^b$
$\rightarrow 5^{\log_5 (x*(x+1))} = 5^{\log_5 (20)}$	Recall $5^{\log_5(\text{anything})}$ = anything
$\rightarrow x^2 + x = 20$	
$\rightarrow x^2 + x - 20 = 0$	Factor
$\rightarrow (x + 5)(x - 4) = 0$	

Solve $(x + 5) = 0$ and $(x - 4) = 0 \rightarrow$ $x = -5$ or $x = 4$

IMPORTANT: always check your answers – put them back into the original equation

$\log_5(4) + \log_5(4+1) = \log_5(20)$ this works.
$\log_5(-5) + \log_5(-5+1) = \log_5(20)$ this does NOT work, you cannot take $\log_5(-5)$.

So the only answer is **$x = 4$**

52. For what value of *x* is it true that $(\log x)^3 = 3*\log(x)$?

Be careful on this one – it is not that obvious.

$(\log x)^3 = 3*\log(x)$	Divide both sides by log(x)
$(\log x)^2 = 3$	Take the square root of both sides
$\log(x) = 3^{1/2}$	Use the property: if a = b then $10^a = 10^b$
$10^{\log(x)} = 10^{\sqrt{3}}$	Recall log is the same thing as $\log_{10}$.
x = $10^{\sqrt{3}}$	

56. A man invests $6500 in an account that pays 6% interest per year, compounded continuously.
56 a. What is the amount after 2 years?

Recall:
<u>Continuously Compounded Interest</u>

$$A(t) = Pe^{r*t}$$

where $A(t)$ = amount after t years.
P = principal (or initial amount).
r = interest rate <u>per year</u>.
t = time in <u>years</u>.

For this we are trying to find A(2). P = 6500, r = 0.06, t = 2

$A(2) = 6500*e^{0.06 * 2}$ ~= 7328.73

So it is worth about **$7,328.73** after 2 years.

56 b. How long will it take for the amount to be $8000?

Here we are given A(t) = 8000 and we need to find t. P = 6500, r = 0.06

$8000 = 6500*e^{0.06 * t}$	Isolate the e by dividing both sides by 6500
$8000/6500 = e^{0.06 * t}$	Take the natural log of both sides
$\ln(8000/6500) = \ln(e^{0.06 * t})$	Simplify
$\ln(8000/6500) = 0.06*t$	Divide both sides by 0.06

$$\frac{\ln\left(\frac{8000}{6500}\right)}{0.06} = t$$

So t is about **3.46 years**.

62. Find the annual percentage yield for an investment that earns 5½% per year, compounded continuously.

For this we are trying to find A(t). P = P, r = 0.055, t = 1
Recall:
<u>Continuously Compounded Interest</u>

$$A(t) = Pe^{r*t}$$

where $A(t)$ = amount after t years.
P = principal (or initial amount).
r = interest rate <u>per year</u>.
t = time in <u>years</u>.

AND notice this section tells you that the formula for
<u>simple interest</u> is: A = P(1 + a.p.y), where a.p.y. = annual percentage yield

So we will take our first equation: $A(t) = Pe^{r*t}$ and put r = 0.055 and t = 1 into it:

$A = Pe^{0.055 * 1} = Pe^{0.055}$

And we set that equal to P*(1 + a.p.y.) and solve for a.p.y.

$Pe^{0.055}$ = P(1 + a.p.y.) divide both sides by P
$e^{0.055}$ = (1 + a.p.y.)
1.05654 ~= (1 + a.p.y.) subtract 1 from both sides
0.05654 ~= a.p.y.

So we conclude that **the annual percentage yield is about 5.65%**.