Section 4.5
Solutions and Hints

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for the book:
Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

This section contains quite a few important equations. Depending on your professor you may need to memorize some, all or none of them. They are restated here just for reference. Solved problems follow.

Exponential Growth Model:

Notice that exponential growth is almost exactly the same as continuously compounded:

Continuously Compounded Interest	Exponential Growth Model
$A(t) = Pe^{r*t}$ where $A(t)$ = amount after t years. P = principal (or initial amount). r = interest rate per year. t = time in years.	$n(t) = n_0e^{r*t}$ where $n(t)$ = population after t years. n_0 = initial population size. r = growth rate per time unit. t = time in time units.

So the only real difference is that the growth model does NOT require time to be measured in years.

Radioactive Decay Model:

$$m(t) = m_0e^{-rt} = m_0e^{-\left(\frac{\ln 2}{h}\right)*t}$$

where $m(t)$ = the mass remaining after time t
m_0 = the initial mass
t = the time
h = the half life
$r = (\ln 2) / h$

Newton's Cooling Law:

$$T(t) = T_s + D_0e^{-kt}$$

where $T(t)$ = the temperature of an object after time t.
T_s = the temperature of the object's surrounding environment
D_0 = the initial temperature difference between the object and its surroundings
t = time
k = some positive constant dependent on the type of object.

pH scale:
The pH of a substance indicates its acidity.
pH = 7 means neutral. pH < 7 means acidic. pH > 7 means basic.

$pH = -\log[H^+]$

where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter (M).

Richter Scale:

$$M = \log(I / S)$$

where M = magnitude of an earthquake
I = intensity of the earthquake (measured by amplitude on a seismograph)
S = standard intensity of a quake, amplitude = 1 micron = 10^{-4} cm

So the magnitude of a standard earthquake is log(1) = 0.

Decibel Scale:

$$\beta = 10*\log(I / I_0)$$

where β = the intensity level of a sound measured in decibels (dB)
$I_0 = 10^{-12}$ watts per square meter (W / m^2) at a frequency of 1000 hertz.
I = the measured intensity of a sound in W / m^2.

6. The frog population in a small pond grows exponentially. The current population is 85 frogs and the relative growth rate is 18% per year.

6a. Find a function that models the population after *t* years.

$n(t) = n_0 e^{r*t}$

where $n(t)$ = population after t years.
n_0 = initial population size.
r = growth rate <u>per time unit</u>.
t = time in <u>time units</u>.

So here $n_0 = 85$
$r = 0.18$ per year
$t = t$ years

And we have our model function: $\mathbf{n(t) = 85*e^{0.18*t}}$

6b. Find the project frog population in 3 years.

Put 3 in for t into the equation of part (a):

$n(3) = 85*e^{0.18 * 3} = 85 * e^{0.54} \sim= 145.86$

So in three years there will be about **145 frogs**.

6c. Find the number of years required for the frog population to reach 600.

Here we are given $n(t) = 800$ and we must find t:

$800 = 85*e^{0.18*t}$	Isolate the e term by dividing both sides by 85
$800/85 = e^{0.18*t}$	Take the natural log of both sides
$\ln(800 / 85) = \ln(e^{0.18*t})$	Simplify
$\ln(800 / 85) = 0.18 * t$	Divide both sides by 0.18

$$\frac{\ln\left(\frac{800}{85}\right)}{0.18} = t$$

Thus t is about **12.455 years**.

18. Radium 221 has a half life of 30 seconds. How long will it take for 95% of a sample to decay?

Apply:

$$m(t) = m_0 e^{-rt} = m_0 e^{-\left(\frac{\ln 2}{h}\right)*t}$$

where $m(t)$ = the mass remaining after time t
m_0 = the initial mass
t = the time
h = the half life
$r = (\ln 2) / h$

We will let our initial amount $m_0 = 1$
and thus our final amount will be m(t) = 0.05 (because 0.95 of it is gone).
h = 30 seconds
t = what we need to solve for (in seconds)

$$0.05 = 1 * e^{-\left(\frac{\ln 2}{30}\right)*t}$$ Take ln of both sides

$$\ln(0.05) = \ln\left(e^{-\left(\frac{\ln 2}{30}\right)*t}\right)$$ Simplify

$$\ln(0.05) = -\left(\frac{\ln 2}{30}\right)*t$$ Divide both sides by –(ln2) / 30

$$\frac{-30 * \ln(0.05)}{\ln 2} = t$$ And t comes out to be about **129.66 seconds**.