Section 6.1
Solutions and Hints

by Brent M. Dingle

for the book:
Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

If you remember nothing else from this section remember:

arc length = radius * angle

$s = r * \theta$

where the angle, θ, is measured in radians.

There are other formulas, but that one is pretty important.

48. A circular arc of length 3 feet subtends a central angle of 25°. Find the radius of the circle.

Start with $s = r*\theta$, $s = 3$ feet, $\theta = 25° = 25*(\pi/180) = (5/36)\pi$ radians

$3 = r * (5/36)\pi \rightarrow 3*(36/5) = r*\pi \rightarrow 21.6 = \pi*r \rightarrow$ **$r = 21.6/\pi$ feet**

52. Memphis, Tennessee and New Orleans Louisiana lie approximately on the same meridian. Memphis has latitude 35° N and New Orleans 30° N. Find the distance between the cities, given the radius of the earth is 3960 miles.

Again start with $s = r*\theta$,

with $r = 3960$ miles and $\theta = 35 - 30 = 5° = 5*(\pi/180) = (1/36)\pi$.

and we need to find s = arc length = distance between the cities.

$s = 3960*(1/36)\pi =$ **110π miles**.

60. A sector of a circle of radius 24 miles has an area of 288 square miles. Find the central angle of the sector.

For this you need a new formula.

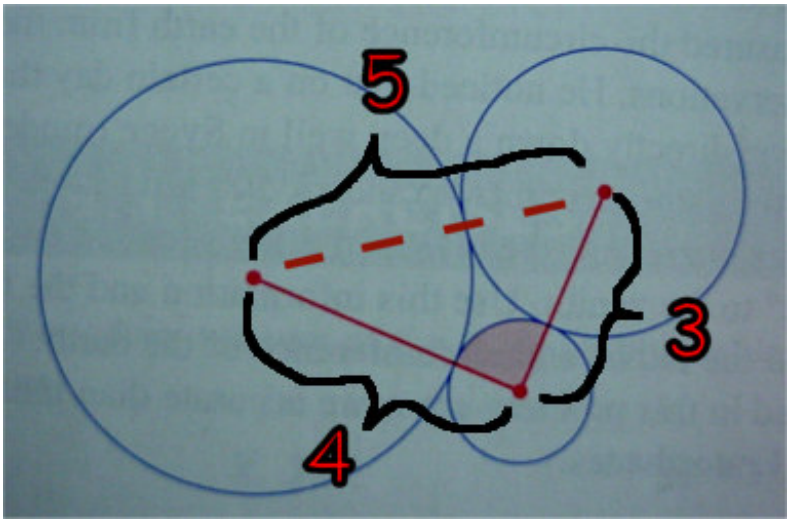
The **area of a sector of circle** = $A = \frac{1}{2} * r^2 * \theta$,
where θ is the central angle of the sector measured in radians
and r of course is the radius of the circle.

For this problem r = 24 miles, A = 288 sq. miles and we need to find θ.

$288 = (\frac{1}{2}) * 24^2 * \theta \rightarrow 288 = 288 * \theta \rightarrow$ **1 radian = θ** (or about 57.3°)

62. Three circles with radii 1, 2 and 3 feet are externally tangent to one another. Find the area of the sector of the circle of radius 1 that is cut off by the line segments joining the center of that circle to the centers of the other two circles.

So you start out with:



Notice you know the length of ALL the sides of the triangle, because you know the radius of each circle. From this you might discern that: $5^2 = 3^2 + 4^2$. Thus you have the (length of the hypotenuse)^2 = (length of side A)^2 + (length of side B)^2. And from that you may conclude the triangle is a right triangle, or rather the angle we are interested in is 90°. Thus we use the area formula given in the text:
The **area of a sector of circle** = $A = \frac{1}{2} * r^2 * \theta$,

For this problem r = 1 foot, $\theta = 90° = \pi/2$, and we need to find A

$A = \frac{1}{2} * 1^2 * \pi/2 =$ **$\pi/4$ square feet**.

66. The earth rotates about its axis once every 23 hours 56 minutes and 4 seconds. The radius of the earth is 3960 miles. Find the linear speed of a point on the equator in miles per hour (mph).

First convert 23 hours 56 minutes and 4 seconds to seconds (I like integers):

23*3600 + 56*60 + 4 = 86164 seconds.

86164 seconds / 3600 seconds per hour = 21541/900 hours

Recall that a complete rotation would be an angle $\theta = 2\pi$.
So the distance a point on the earth moves (rotationally) in 1 rotation = the circumference of the earth = $2\pi r = 2\pi * 3960$ miles.

And the points linear speed = $(2\pi * 3960 \text{ miles}) / 86164 \text{ seconds}$
$= (2\pi * 3960 \text{ miles}) / (21541/900) \text{ hours}$
$= (7128000 / 21541)*\pi \text{ miles per hour}$
$\sim= 330.9038578*\pi \text{ miles per hour}$
$\sim=$ **1039.565129 miles per hour**