Section 6.4
Solutions and Hints

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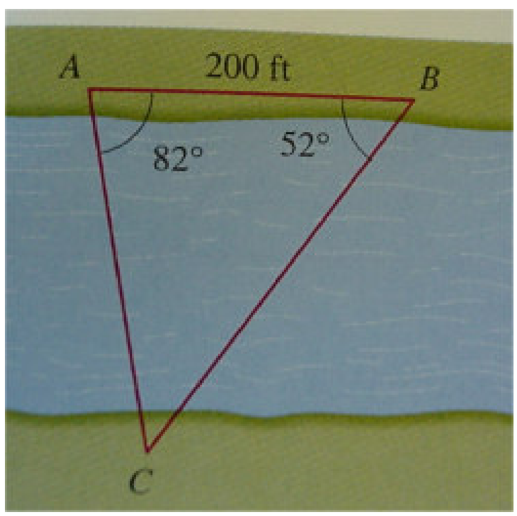
for the book:

Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

23. To find the distance across a river, a surveyor chooses points A and B, which are 200 feet apart on one side of the river. He then chooses a reference point C on the opposite side of the river and finds that $\angle$ BAC = 82° and $\angle$ ABC = 52°. Approximate the distance from A to C.

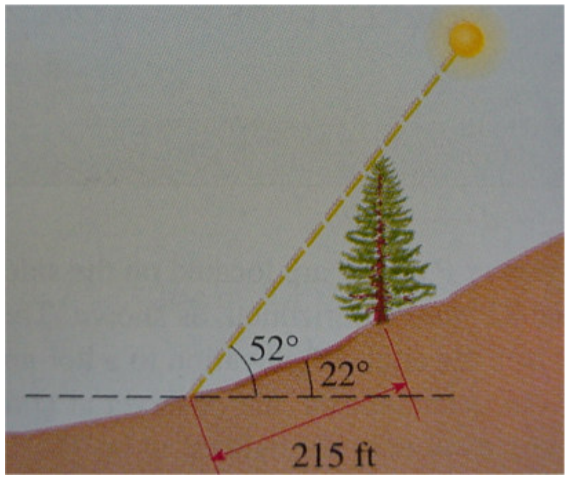


Recall the sum of the angles of a triangle must be 180°.
Thus $\angle$ ACB = 180 – (82+52) = 46°.
Let c = 200 feet
Let b = distance from A to C

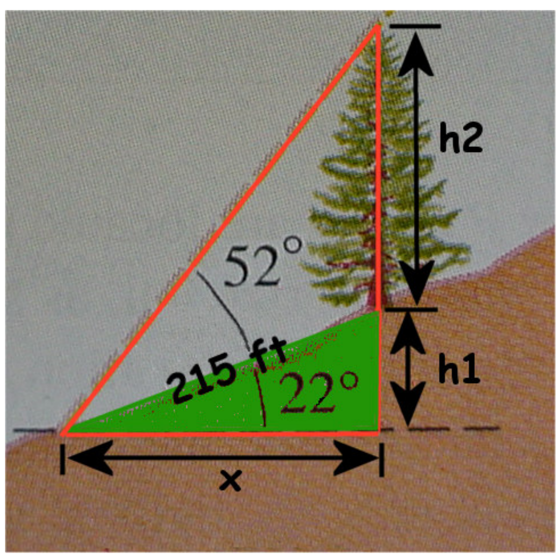
From the Law of Sines we have:
(sin C) / c = (sin B) / b → sin(46°) / 200 = sin(52°) / b
→ b* sin(46°) / 200 = sin(52°)
→ b* sin(46°) = 200*sin(52°)
→ b = 200*sin(52°) / sin(46°)
→ b ≅ **219 feet**

26. A tree on a hillside casts a shadow 215 feet down the hill. If the angle of inclination of the hillside is known to be 22° to the horizontal and the angle of elevation to the sun is 52°, find the height of the tree.

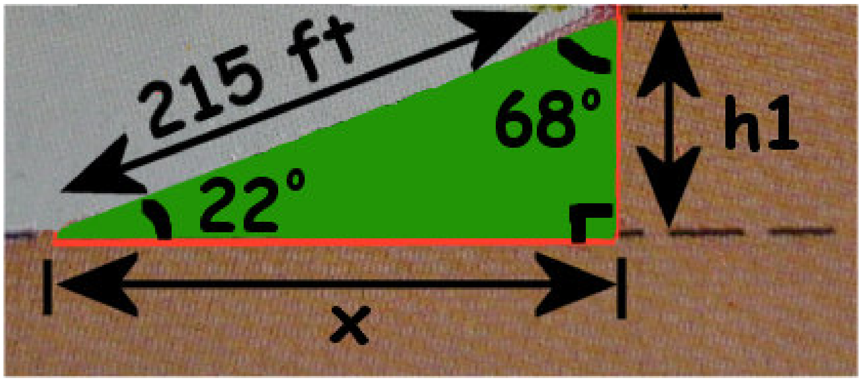
Assume the tree is growing perpendicular to the horizontal (i.e. straight upwards).



Notice there are really 2 triangles in this picture of significance:



So looking at the green triangle we will find x and h1
Notice all angles are known as the sum of the angles of a triangle = 180°.



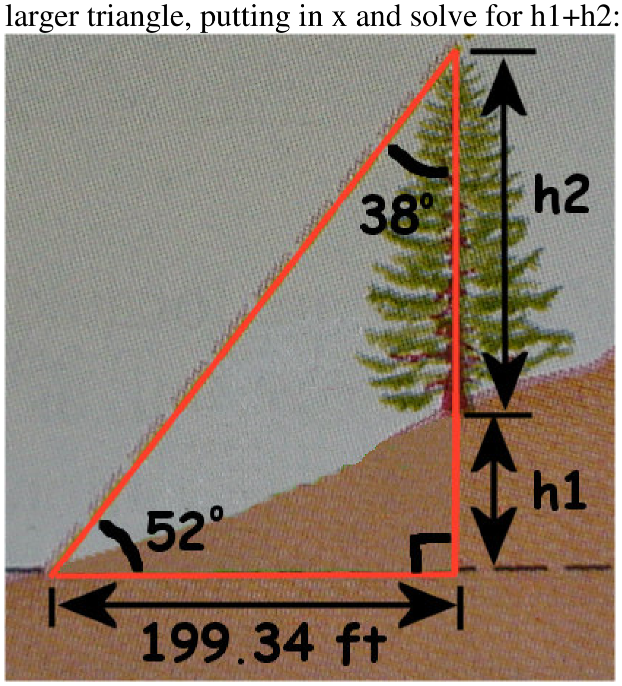
Finding x:

$\sin(68°) / x = \sin(90°)/215$ → $\sin(68°) = x * \sin(90°)/215$
→ $215*\sin(68°) = x*\sin(90°)$
→ $215*\sin(68°) = x*1$
→ $199.34 \cong x$

Finding h1:

$\sin(22°) / h1 = \sin(90°)/215$ → $\sin(22°) = h1 * \sin(90°)/215$
→ $215*\sin(22°) = h1 * \sin(90°)$
→ $215* \sin(22°) = h1 * 1$
→ $80.54 \cong h1$

Now we will use the larger triangle, putting in x and solve for h1+h2:



Thus by Law of Sines:

$\sin(52°) / (h1 + h2) = \sin(38°) / 199.34$

$\sin(52°) = (h1 + h2)*\sin(38°) / 199.34$

$199.34*\sin(52°) = (h1 + h2)*\sin(38°)$

$199.34*\sin(52°) / \sin(38°) = (h1 + h2)$

$255.14 \cong (h1 + h2)$

And we previously determined that $h1 \cong 80.54$, so

$255.14 \cong 80.54 + h2$

$h2 \cong$ **174.6 feet is the approximate height of the tree**.