# Section 7.3
# Solutions and Hints

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## for the book:

**Precalculus, Mathematics for Calculus 4th Edition**
**by James Stewart, Lothar Redlin and Saleem Watson.**

Depending on your professor you may or may not need to memorize a large number of formulas. Regardless almost all the professors will expect you to know the following:

| | |
|---|---|
| $\cos(0) = 1$ | $\sin(0) = 0$ |
| $\cos(\pi/6) = \frac{\sqrt{3}}{2}$ | $\sin(\pi/6) = ½$ |
| $\cos(\pi/4) = \frac{\sqrt{2}}{2}$ | $\sin(\pi/4) = \frac{\sqrt{2}}{2}$ |
| $\cos(\pi/3) = ½$ | $\sin(\pi/3) = \frac{\sqrt{3}}{2}$ |
| $\cos(\pi/2) = 0$ | $\sin(\pi/2) = 1$ |

Notice that $0 < \pi/6 < \pi/4 < \pi/3 < \pi/2$, so the angles as listed are in order. Also notice that on the unit circle the x coordinates = cosine values and the y coordinates = sine values.

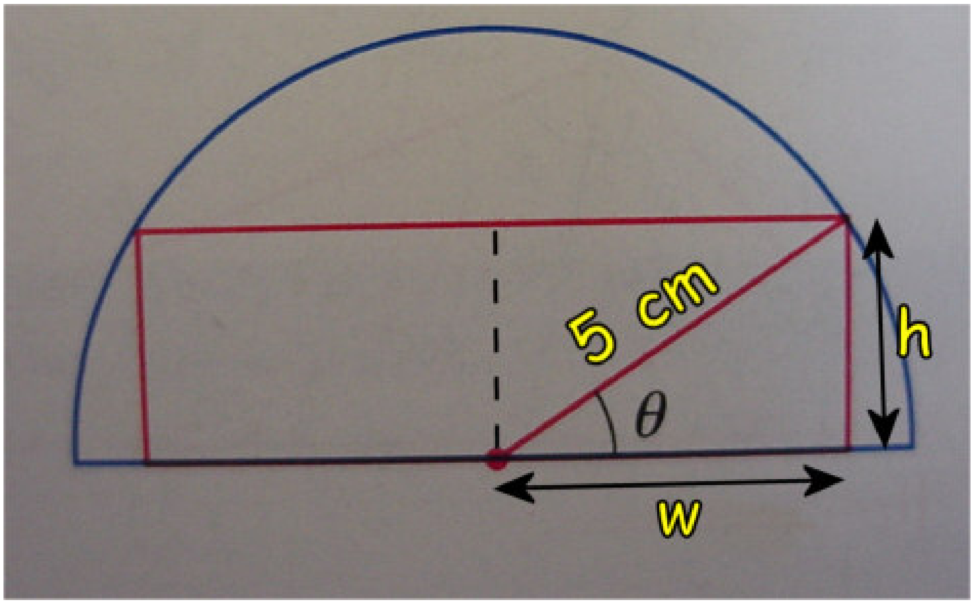
**52. Find the value of the product or sum:**

Recall: $\cos(x) + \cos(y) = 2*\cos((x+y)/2)*\cos((x - y)/2)$

$$\cos(\pi/12) + \cos(5\pi/12) = 2*\cos((\pi/12 + 5\pi/12)/2)*\cos((\pi/12 - 5\pi/12)/2)$$
$$= 2 * \cos((6\pi/12)/2) * \cos((4\pi/12)/2)$$
$$= 2*\cos(\pi/4) * \cos(\pi/6)$$
$$= 2*\frac{\sqrt{2}}{2}*\frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{6}}{2}$$

**80. A rectangle is to be inscribed in a semicircle of radius 5 cm.**

**80a. Show that the area of the rectangle is modeled by the function:**
**$A(\theta) = 25 * \sin(2\theta)$**




Notice the hypotenuse of the triangle must always equal 5 cm.
Also notice we added the h and w variables into the picture.

You should see that there are 2 rectangles of height = h and width = w.
*You should also know that for the rectangle to be inscribed in a semicircle,*
*$\theta$ will always range in value such that $0° < \theta < 90°$*

With this in mind the area of the rectangle = (w * h) + (w * h)   (2 rectangles)

We now need to find an equations such that h and w are expressed in terms of θ:

$\sin(\theta)$ = opposite / hypotenuse = h / 5 → $h = 5*\sin(\theta)$
$\cos(\theta)$ = adjacent / hypotenuse = w / 5 → $w = 5*\cos(\theta)$

Area = $A = 2*h*w$ $= 2 * (5*\sin\theta) * (5*\cos\theta)$
$= 25 * (2 * \sin\theta * \cos\theta)$,  which by double angle formula is:
$= 25 * \sin(2\theta)$

**80b. Find the largest possible area for the inscribed rectangle.**

A= 25*sin(2θ) will be maximum when sin(2θ) is maximum.

Recall that sin( ) achieves its maximum at 90° = π/2.
So set 2θ = π/2 → θ = π/4 = 45°.

Thus the maximum area is 25*sin(π/4) = $25 * \frac{\sqrt{2}}{2} \cong$ **17.68 $cm^2$**.

**80c. Find the dimensions of the inscribed rectangle with the largest area.**

Recall from part (a):

h = 5*sin(θ)
w = 5*cos(θ)

And from part (b) θ = π/4 when the rectangle's area is greatest. So

$$h = 5*\sin(\pi/4) = 5 * \frac{\sqrt{2}}{2}$$

$$w = 5*\cos(\pi/4) = 5 * \frac{\sqrt{2}}{2}$$

But recall from the picture that the full width of the rectangle = 2*w.

Thus the dimensions of the rectangle are: $5 * \frac{\sqrt{2}}{2}$ by $5 * \sqrt{2}$

Or roughly: **3.54 cm by 7.07 cm**