Section 7.7
Solutions and Hints

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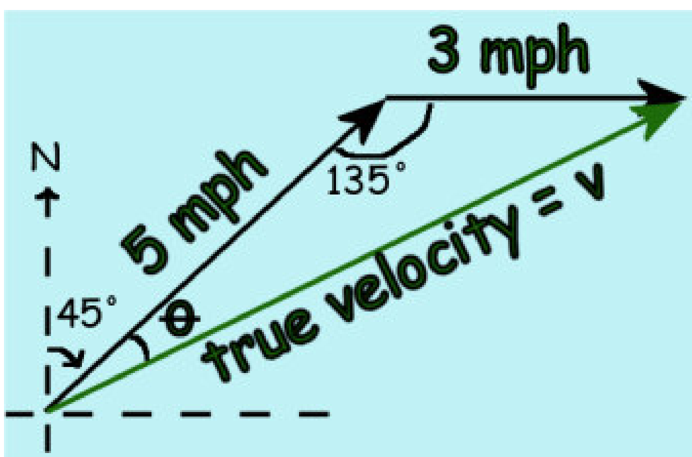
for the book:
<u>Precalculus, Mathematics for Calculus 4th Edition</u>
by James Stewart, Lothar Redlin and Saleem Watson.

If you are going on to higher level math classes or physics classes, do as many of these problems as possible.

42. A migrating salmon heads in the direction N 45° E, swimming at 5 mph relative to the water. The prevailing ocean currents flow due east at 3 mph. Find the true velocity of the fish.



Here we need to find the magnitude v of the velocity as well as its direction, preferably in terms of the indicated θ (thus we will be able to say v mph at N $(45 + \theta)$° E.

There are several ways to approach this problem. I will present two methods:

Method 1:

The first method is going to deviate from the book's suggested methods to refresh some of the previous sections:

To find *v* we can use the Law of Cosines:

$v^2 = 5^2 + 3^2 - 2*5*3*\cos(135°)$

$v^2 = 34 + 15\sqrt{2}$

$v = \pm\sqrt{34 + 15\sqrt{2}}$ *and we only take the positive result* $\rightarrow$ $v \cong 7.43$ mph

To find θ we can use the Law of Sines:

$$\frac{\sin(135)}{7.43} = \frac{\sin(\theta)}{3}$$

$\rightarrow 3*\sin(135°) / 7.43 = \sin(\theta)$

$\rightarrow \sin^{-1}(3*\sin(135°) / 7.43) = \theta$ $\rightarrow \theta \cong 16.5879°$

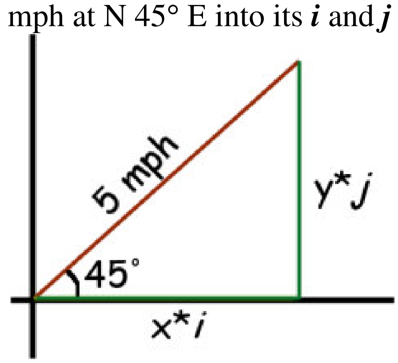
Recall we need to add 45° to θ → 61.5879°

So the salmon is actually moving at **7.43 mph in the direction N 61.5879° E**

Method 2:

This goes more along the line of this section.

First we must break 5 mph at N 45° E into its ***i*** and ***j*** components:



So we get $\cos(45°) = x / 5 \rightarrow x = \frac{5\sqrt{2}}{2}\boldsymbol{i}$.

And $\sin(45°) = y / 5 \rightarrow y = \frac{5\sqrt{2}}{2}\boldsymbol{j}$.

So the velocity vector of the fish is: $< \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} >$

And clearly 3 mph due east = 3***i***, so its velocity vector is: < 3, 0 >

Adding these two vectors we get:

$$< \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} > + < 3, 0 > \rightarrow < \frac{6 + 5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} >$$

Notice $\sqrt{\left(\frac{6+5\sqrt{2}}{2}\right)^2+\left(\frac{5\sqrt{2}}{2}\right)^2} \cong 7.43$ mph

Let α = the degree measured from the x-axis (so it would be E α N)

tan(α) = y / x = $\frac{5\sqrt{2}}{2} \,/\, \frac{6+5\sqrt{2}}{2} \cong 0.54097 \rightarrow \alpha \cong 28.412°$

And we need it to be N α E, so we say θ = 90 - α → θ = 61.5879°

And we would conclude:

the salmon is traveling about 7.43 mph at N 61.5879° E

Which is the same conclusion as we arrived at using the first method !

Notice for this section the answer may be expressed as:

$$v = \frac{6+5\sqrt{2}}{2}\boldsymbol{i} + \frac{5\sqrt{2}}{2}\boldsymbol{j}.$$

50. A woman walks due west on the deck of an ocean liner at 2 mph. The ocean liner is moving due north at a speed of 25 mph. Find the speed and direction of the woman relative to the surface of the water.

The woman's velocity vector is <-2, 0>
The ocean liner's velocity vector is <0, 25>
Assume the water is considered not moving (no current).
If she stood still here velocity relative to the water would be 25 mph N.
If the boat stopped moving her velocity relative to the water would be –2 mph W.

So the woman's velocity relative to the water should be:

<-2, 0> + <0, 25> = <-2, 25>

or

-2i + 25j

Which gives a magnitude of $((-2)^2 + 25^2)^{1/2} \cong 25.0798$ mph

You might also let α = the angle of movement measured from the x-axis, so:

tan(α) = opposite / adjacent = 25/2 *(make her walk, then move the boat)*

$\alpha = \tan^{-1}(25/2) \rightarrow \alpha \cong 85.426°$ this is in terms of W α N but we want N α W

θ = 90 - α ≅ 4.573°

Thus we could also say **she is moving at about 25.0798 mph heading N 4.573° W**