

Section 8.1

Solutions and Hints

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for the book:

Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

24. Find all solutions to the system of equations:

Eq 1. $x + \sqrt{y} = 0$ \leftarrow this is ugly so multiply both sides by $x - \sqrt{y}$
Eq 2. $y^2 - 4x^2 = 12$

Eq 1. $x^2 - y = 0$ Now we can start solving the system
Eq 2. $y^2 - 4x^2 = 12$

$$\begin{array}{r} 4*(x^2 - y = 0) \\ + y^2 - 4x^2 = 12 \\ \hline \end{array}$$

Multiply eq.1 by 4 and add it to eq. 2

$$y^2 - 4y = 12$$

Now solve for y

$$\begin{array}{l} y^2 - 4y - 12 = 0 \\ (y - 6)(y + 2) = 0 \end{array}$$

Factor

$y = 6$ or -2 put this into $x + \sqrt{y} = 0$ gives $x = -\sqrt{6}$ or $x = -\sqrt{-2} = -i\sqrt{2}$

So the solutions are:

$$\begin{array}{l} x = -\sqrt{6} \text{ and } y = 6 \\ x = -i\sqrt{2} \text{ and } y = -2 \end{array}$$

42. A right triangle has an area of 84 sq. ft and a hypotenuse of length 25 ft. What are the lengths of its other two sides?

For this we will use 2 equations:

$$\text{Area of a triangle, } A = \frac{1}{2} a * b$$

$$\text{Pythagorean Theorem: } a^2 + b^2 = c^2$$

For this problem $A = 84$ feet and $c = 25$ feet. We need to find a and b .

$$\text{Equation 1: } \frac{1}{2} * a * b = 84$$

$$\text{Equation 2: } a^2 + b^2 = 625 \quad (25^2 = 625)$$

Using equation 1, we will solve for b in terms of a :

$$\frac{1}{2} * a * b = 84 \rightarrow ab = 168 \rightarrow b = 168/a$$

We now put $168/a$ in for b into equation 2:

$$a^2 + b^2 = 625 \rightarrow a^2 + (168/a)^2 = 625$$

$$\rightarrow a^2 + \frac{168^2}{a^2} = 625, \quad \text{Multiply both sides by } a^2$$

$$\rightarrow a^4 + 28224 = 625a^2, \quad \text{Subtract everything to the left side}$$

$$\rightarrow a^4 - 625a^2 + 28224 = 0, \quad \text{Factor}$$

$$\rightarrow (a^2 - 576)(a^2 - 49) = 0$$

Which means $a^2 = 576$ or $a^2 = 49$, notice we will only take the positive results.

$$a = \sqrt{576} = 24 \quad \text{or} \quad a = \sqrt{49} = 7$$

And using $b = 168/a$ we then get

$$b = 168/24 = 7 \quad \text{or} \quad b = 168/7 = 24$$

So the possible solutions are:

$a = 24$ and $b = 7$ or $a = 7$ and $b = 24$
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