Section 9.2
Solutions and Hints

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for the book:
Precalculus, Mathematics for Calculus 4th Edition
by James Stewart, Lothar Redlin and Saleem Watson.

You will need to know the equation of ellipses centered at the origin and what all the variables mean (see page 736 of your book).

The general equation of an ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

If the denominator below the x^2 is > that below the y^2 (as written if $a^2 > b^2$)
then the major axis will be horizontal (with vertices (-a, 0) and (a, 0))
If the denominators are equal
then you have a circle.
If the denominator below the y^2 is > that below the x^2 (as written if $a^2 < b^2$)
then the major axis will be vertical (with vertices (0, -b) and (0, b))

For ellipses with a horizontal major axis:
The foci will be at (-c, 0) and (c, 0)
where c^2 = (denom below $x^2)^2$ – (denom below $y^2)^2$

For ellipses with a vertical major axis:
The foci will be at (0, -c) and (0, c)
where c^2 = (denom below $y^2)^2$ – (denom below $x^2)^2$

36. Find an equation for the ellipse that satisfies the conditions: Endpoints of minor axis (0, 3) and (0, -3), distance between foci 8

Notice the distance between foci would be = c – (-c) = 2c.

So 2c = 8 → c = 4.

With the endpoints of the minor axis being (0, 3) and (0, -3) we know the major axis is horizontal. Thus following the book's notation on page 736 we have the general form of the ellipse as being:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

And we have $b^2 = 3*3 = 9$

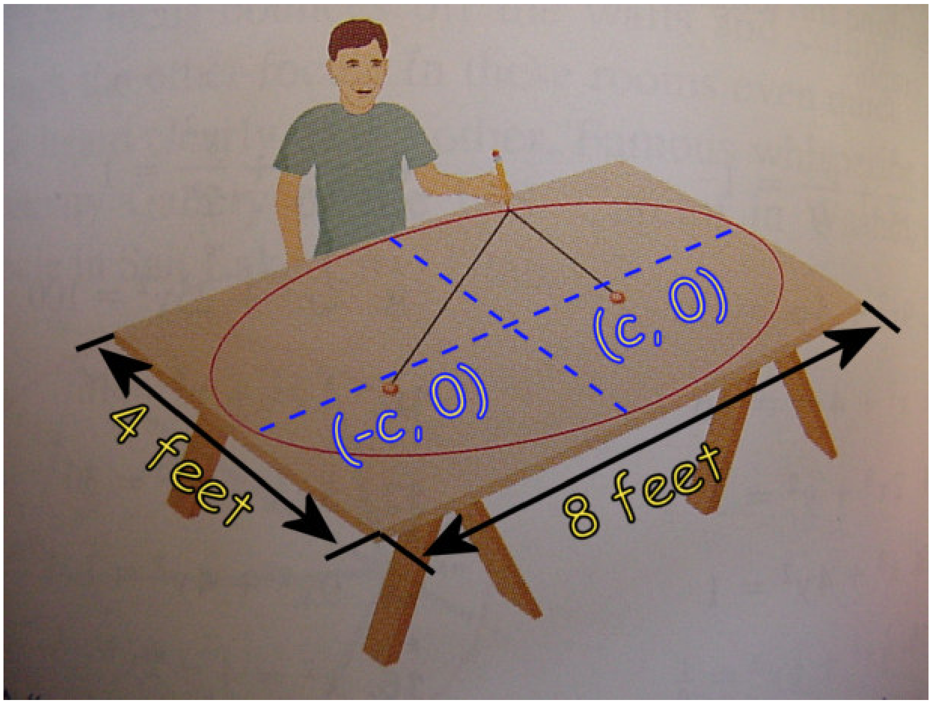
Further we know by definition: $c^2 = a^2 - b^2$, and we know c = 4 from above.

Thus $4^2 = a^2 - 9$ → $16+9 = a^2$
→ $25 = a^2$
→ $5 = a$ (we only care about the positive result)

So the equation of the ellipse (putting a^2 and b^2 in) is:

Equation of ellipse = $\frac{x^2}{25} + \frac{y^2}{9} = 1$

46. A carpenter wishes to construct an elliptical table top from a sheet of plywood, 4 foot by 8 foot. He will trace out the ellipse using the "thumbtack and string" method. What length of string should be used and how far apart should the tacks be placed to create the largest possible ellipse?



So effectively we are given that:
The endpoints of the major axis are (-8/2, 0) and (8/2,0) = (-4, 0) and (4, 0)
The endpoints of the minor axis are (0, -4/2) and (0, 4/2) = (0, -2) and (0, 2)

Thus we have $a^2 = 4*4 = 16$ and $b = 2*2 = 4$

The tacks need to be placed at the foci of the ellipse, so at (-c, 0) and (c, 0)
We know $c^2 = a^2 - b^2 \rightarrow c^2 = 16 - 4$

$\rightarrow c = \pm\sqrt{12} \cong 3.4641$ feet $\cong$ 3 feet 5 and 9/16 inches

And the string length must be able to reach from the negative focus, to the positive vertex and back to the positive focus (from (-c, 0) to (a, 0) back to (c, 0)):
The distance from (-c, 0) to (a, 0) = (c + a) and the distance from (a, 0) to (c, 0) = (a – c)
Thus the string length = (c + a) + (a – c) = 2a = 2*4 = 8 feet.
And the complete answer is:

From the board's center point, place each tack 3 feet 5 and 9/16 inches towards the short sides of the board (roughly 7 feet apart). The length of the needed string is 8 feet.